Chapter 1 Summary, Review, and Test

Summary

DEFINITIONS AND CONCEPTS	EXAMPLES
1.1 Graphs and Graphing Utilities	
a. The rectangular coordinate system consists of a horizontal number line, the x-axis, and a vertical number line, the y-axis, intersecting at their zero points, the origin. Each point in the system corresponds to an ordered pair of real numbers (x, y). The first number in the pair is the x-coordinate; the second number is the y-coordinate. See Figure 1.1 on page 136.	Ex. 1, p. 137
b. An ordered pair is a solution of an equation in two variables if replacing the variables by the corresponding coordinates results in a true statement. The ordered pair is said to satisfy the equation. The graph of the equation is the set of all points whose coordinates satisfy the equation. One method for graphing an equation is to plot ordered-pair solutions and connect them with a smooth curve or line.	Ex. 2, p. 138; Ex. 3, p. 138
c. An x-intercept of a graph is the x-coordinate of a point where the graph intersects the x-axis. The y-coordinate corresponding to an x-intercept is always zero. A y-intercept of a graph is the y-coordinate of a point where the graph intersects the y-axis. The x-coordinate corresponding to a y-intercept is always zero.	Ex. 5, p. 141
1.2 Basics of Functions and Their Graphs	
a. A relation is any set of ordered pairs. The set of first components is the domain of the relation and the set of second components is the range.	Ex. 1, p. 147
b. A function is a correspondence from a first set, called the domain, to a second set, called the range, such that each element in the domain corresponds to exactly one element in the range. If any element in a relation's domain corresponds to more than one element in the range, the relation is not a function.	Ex. 2, p. 150
c. Functions are usually given in terms of equations involving x and y, in which x is the independent variable and y is the dependent variable. If an equation is solved for y and more than one value of y can be obtained for a given x, then the equation does not define y as a function of x. If an equation defines a function, the value of the function at x, $f(x)$, often replaces y.	Ex. 3, p. 151; Ex. 4, p. 152
d. The graph of a function is the graph of its ordered pairs.	Ex. 5, p. 153
e. The vertical line test for functions: If any vertical line intersects a graph in more than one point, the graph does not define y as a function of x.	Ex. 6, p. 154
f. The graph of a function can be used to determine the function's domain and its range. To find the domain, look for all the inputs on the x-axis that correspond to points on the graph. To find the range, look for all the outputs on the y-axis that correspond to points on the graph.	Ex. 8, p. 157
g. The zeros of a function, f, are the values of x for which $f(x) = 0$. At the real zeros, the graph of f has x-intercepts. A function can have more than one x-intercept but at most one y-intercept.	Figure 1.26, p. 159
1.3 More on Functions and Their Graphs	
a. A function is increasing on intervals where its graph rises, decreasing on intervals where it falls, and constant on intervals where it neither rises nor falls. Precise definitions are given in the box on page 165.	Ex. 1, p. 166
b. If the graph of a function is given, we can often visually locate the number(s) at which the function has a relative maximum or relative minimum. Precise definitions are given in the box on page 166.	Figure 1.28, p. 167
c. The graph of an even function in which $f(-x) = f(x)$ is symmetric with respect to the y-axis. The graph of an odd function in which $f(-x) = -f(x)$ is symmetric with respect to the origin.	Ex. 2, p. 167
d. Piecewise functions are defined by two (or more) equations over a specified domain. Some piecewise functions are called step functions because their graphs form discontinuous steps. An example is $f(x) = \text{int}(x)$, where $\text{int}(x)$ is the greatest integer that is less than or equal to x.	Ex. 3, p. 169; Ex. 4, p. 170; Figure 1.34, p. 171
e. The difference quotient of a function f is $\dfrac{f(x+h) - f(x)}{h}, h \neq 0.$	Ex. 5, p. 171

DEFINITIONS AND CONCEPTS	EXAMPLES
1.4 Linear Functions and Slope	
a. The slope, m, of the line through (x_1, y_1) and (x_2, y_2) is $m = \frac{y_2 - y_1}{x_2 - x_1}$.	Ex. 1, p. 179
b. Equations of lines include point-slope form, $y - y_1 = m(x - x_1)$, slope-intercept form, $y = mx + b$, and general form, $Ax + By + C = 0$. The equation of a horizontal line is $y = b$; a vertical line is $x = a$. A vertical line is not a linear function.	Ex. 2, p. 181; Ex. 3, p. 182; Ex. 5, p. 184; Ex. 6, p. 184
c. Linear functions in the form $f(x) = mx + b$ can be graphed using the slope, m, and the y-intercept, b. (See the box on page 183.) Linear equations in the general form $Ax + By + C = 0$ can be solved for y and graphed using the slope and the y-intercept. Intercepts can also be used to graph $Ax + By + C = 0$. (See the box on page 186.)	Ex. 4, p. 183; Ex. 7, p. 185; Ex. 8, p. 186
1.5 More on Slope	
a. Parallel lines have equal slopes. Perpendicular lines have slopes that are negative reciprocals.	Ex. 1, p. 193; Ex. 2, p. 194
b. The slope of a linear function is the rate of change of the dependent variable per unit change of the independent variable.	Ex. 3, p. 196
c. The average rate of change of f from x_1 to x_2 is $$\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$	Ex. 4, p. 197; Ex. 5, p. 198
d. In a finction expresses an object's position, $s(t)$, in terms of time, t, the average velocity of the object from t_1 to t_2 is $$\frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}.$$	Ex. 6, p. 199
1.6 Transformations of Functions	
a. Table 1.3 on page 205 shows the graphs of the constant function, $f(x) = c$, the identity function, $f(x) = x$, the absolute value function, $f(x) = \|x\|$, the standard quadratic function, $f(x) = x^2$, the square root function, $f(x) = \sqrt{x}$, the standard cubic function, $f(x) = x^3$, and the cube root function, $f(x) = \sqrt[3]{x}$. The table also lists characteristics of each function.	
b. Table 1.4 on page 213 summarizes how to graph a function using vertical shifts, $y = f(x) \pm c$, horizontal shifts, $y = f(x \pm c)$, reflections about the x-axis, $y = -f(x)$, reflections about the y-axis, $y = f(-x)$, vertical stretching, $y = cf(x), c > 1$, vertical shrinking, $y = cf(x), 0 < c < 1$, horizontal shrinking, $y = f(cx), c > 1$, and horizontal stretching, $y = f(cx), 0 < c < 1$.	Ex. 1, p. 206; Ex. 2, p. 208; Ex. 3, p. 208; Ex. 4, p. 210; Ex. 5, p. 210; Ex. 6, p. 211; Ex. 7, p. 212
c. A function involving more than one transformation can be graphed in the following order: (1) horizontal shifting; (2) stretching or shrinking; (3) reflecting; (4) vertical shifting.	Ex. 8, p. 214; Ex. 9, p. 215
1.7 Combinations of Functions; Composite Functions	
a. If a function f does not model data or verbal conditions, its domain is the largest set of real numbers for which the value of $f(x)$ is a real number. Exclude from a function's domain real numbers that cause division by zero and real numbers that result in an even root of a negative number.	Ex. 1, p. 221
b. When functions are given as equations, they can be added, subtracted, multiplied, or divided by performing operations with the algebraic expressions that appear on the right side of the equations. Definitions for the sum $f + g$, the difference $f - g$, the product fg, and the quotient $\frac{f}{g}$ functions, with domains $D_f \cap D_g$, and $g(x) \neq 0$ for the quotient function, are given in the box on page 222.	Ex. 2, p. 223; Ex. 3, p. 224
c. The composition of functions f and $g, f \circ g$, is defined by $(f \circ g)(x) = f(g(x))$. The domain of the composite function $f \circ g$ is given in the box on page 226. This composite function is obtained by replacing each occurrence of x in the equation for f with $g(x)$.	Ex. 4, p. 226; Ex. 5, p. 228

DEFINITIONS AND CONCEPTS	EXAMPLES
1.8 Inverse Functions	
a. If $f(g(x)) = x$ and $g(f(x)) = x$, function g is the inverse of function f, denoted f^{-1} and read "f-inverse." Thus, to show that f and g are inverses of each other, one must show that $f(g(x)) = x$ and $g(f(x)) = x$.	Ex. 1, p. 235
b. The procedure for finding a function's inverse uses a switch-and-solve strategy. Switch x and y, and then solve for y. The procedure is given in the box on page 235.	Ex. 2, p. 236; Ex. 3, p. 236; Ex. 4, p. 237
c. The horizontal line test for inverse functions: A function f has an inverse that is a function, f^{-1}, if there is no horizontal line that intersects the graph of the function f at more than one point.	Ex. 5, p. 238
d. A one-to-one function is one in which no two different ordered pairs have the same second component. Only one-to-one functions have inverse functions.	
e. If the point (a, b) is on the graph of f, then the point (b, a) is on the graph of f^{-1}. The graph of f^{-1} is a reflection of the graph of f about the line $y = x$.	Ex. 6, p. 239; Ex. 7, p. 240
1.9 Distance and Midpoint Formulas; Circles	
a. The distance, d, between the points (x_1, y_1) and (x_2, y_2) is given by $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.	Ex. 1, p. 244
b. The midpoint of the line segment whose endpoints are (x_1, y_1) and (x_2, y_2) is the point with coordinates $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.	Ex. 2, p. 245
c. The standard form of the equation of a circle with center (h, k) and radius r is $(x - h)^2 + (y - k)^2 = r^2$.	Ex. 3, p. 246; Ex. 4, p. 247; Ex. 5, p. 247
d. The general form of the equation of a circle is $x^2 + y^2 + Dx + Ey + F = 0$.	
e. To convert from the general form to the standard form of a circle's equation, complete the square on x and y.	Ex. 6, p. 249
1.10 Modeling with Functions	
a. Verbal models are often helpful in obtaining functions from verbal descriptions.	Ex. 1, p. 253; Ex. 2, p. 255
b. Functions can be constructed from formulas, such as formulas for area, perimeter and volume (Table P. 6 on page 106) and formulas for surface area (Table 1.5 on page 259).	Ex. 3, p. 256
c. If a problem's conditions are modeled by a function whose equation contains more than one variable, use the given information to write an equation among these variables. Then use this equation to eliminate all but one of the variables in the function's expression.	Ex. 4, p. 257; Ex. 5, p. 259; Ex. 6, p. 260; Ex. 7, p. 261

Review Exercises

1.1

Graph each equation in Exercises 1–4. Let $x = -3, -2, -1, 0, 1, 2,$ and 3.

1. $y = 2x - 2$

2. $y = x^2 - 3$

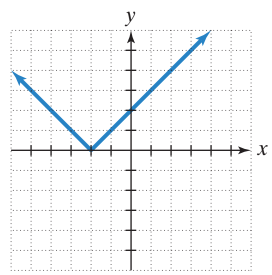
3. $y = x$

4. $y = |x| - 2$

5. What does a $[-20, 40, 10]$ by $[-5, 5, 1]$ viewing rectangle mean? Draw axes with tick marks and label the tick marks to illustrate this viewing rectangle.

In Exercises 6–8, use the graph and determine the x-intercepts, if any, and the y-intercepts if any. For each graph, tick marks along the axes represent one unit each.

6.



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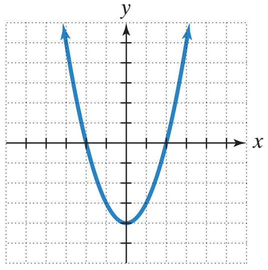

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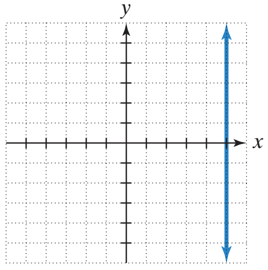
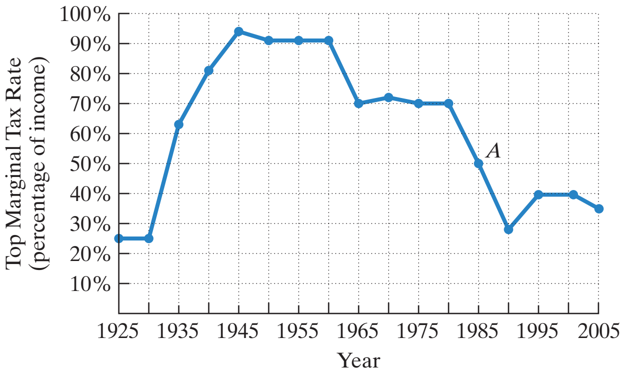


The line graph shows the top marginal income tax rates in the United States from 1925 through 2005. Use the graph to solve Exercises 9–14.

Top United States Marginal Tax Rates, 1925–2005



Source: National Taxpayers Union

9. What are the coordinates of point A? What does this mean in terms of the information given by the graph?

10. Estimate the top marginal tax rate in 2005.

11. For the period shown, during which year did the United States have the highest marginal tax rate? Estimate, to the nearest percent, the tax rate for that year.

12. For the period from 1950 through 2005, during which year did the United States have the lowest marginal tax rate? Estimate, to the nearest percent, the tax rate for that year.

13. For the period shown, during which ten-year period did the top marginal tax rate remain constant? Estimate, to the nearest percent, the tax rate for that period.

14. For the period shown, during which five-year period did the top marginal tax rate increase most rapidly? Estimate, to the nearest percent, the increase in the top tax rate for that period.

1.2 and 1.3

In Exercises 15–17, determine whether each relation is a function. Give the domain and range for each relation.

15. $\{(2, 7), (3, 7), (5, 7)\}$

16. $\{(1, 10), (2, 500), (13, \pi)\}$

17. $\{(12, 13), (14, 15), (12, 19)\}$

In Exercises 18–20, determine whether each equation defines y as a function of x.

18. $2x + y = 8$ **19.** $3x^2 + y = 14$

20. $2x + y^2 = 6$

In Exercises 21–24, evaluate each function at the given values of the independent variable and simplify.

21. $f(x) = 5 - 7x$

a. $f(4)$ **b.** $f(x + 3)$ **c.** $f(-x)$

22. $g(x) = 3x^2 - 5x + 2$

a. $g(0)$ **b.** $g(-2)$

c. $g(x - 1)$ **d.** $g(-x)$

23. $g(x) = \begin{cases} \sqrt{x - 4} & \text{if } x \geq 4 \\ 4 - x & \text{if } x < 4 \end{cases}$

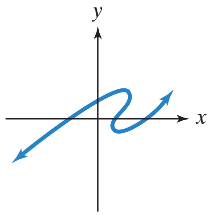
a. $g(13)$ **b.** $g(0)$ **c.** $g(-3)$

24. $f(x) = \begin{cases} \dfrac{x^2 - 1}{x - 1} & \text{if } x \neq 1 \\ 12 & \text{if } x = 1 \end{cases}$

a. $f(-2)$ **b.** $f(1)$ **c.** $f(2)$

In Exercises 25–30, use the vertical line test to identify graphs in which y is a function of x.

25.



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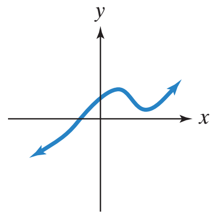

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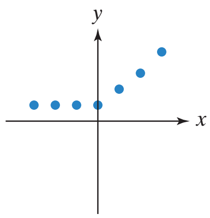

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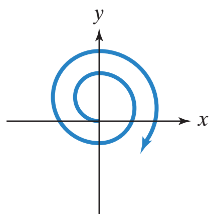

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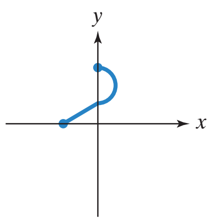

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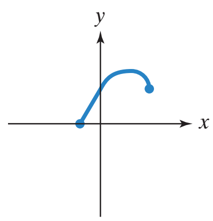
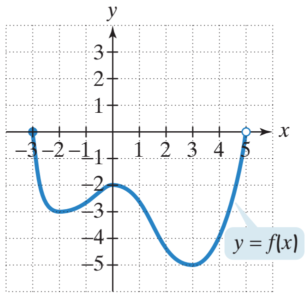


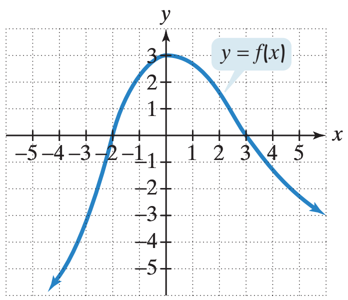
In Exercises 31–33, use the graph to determine **a.** *the function's domain;* **b.** *the function's range;* **c.** *the x-intercepts, if any;* **d.** *the y-intercept, if there is one;* **e.** *intervals on which the function is increasing, decreasing, or constant; and* **f.** *the missing function values, indicated by question marks, below each graph.*

31.



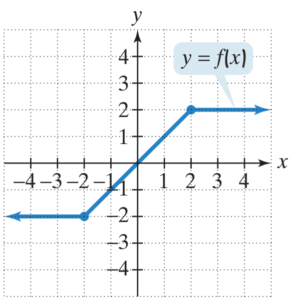
$f(-2) = ?\quad f(3) = ?$

32.



$f(-2) = ?\quad f(6) = ?$

33.



$f(-9) = ?\quad f(14) = ?$

In Exercises 34–35, find each of the following:

a. *The numbers, if any, at which f has a relative maximum. What are these relative maxima?*

b. *The numbers, if any, at which f has a relative minimum. What are these relative minima?*

34. Use the graph in Exercise 31.

35. Use the graph in Exercise 32.

In Exercises 36–38, determine whether each function is even, odd, or neither. State each function's symmetry. If you are using a graphing utility, graph the function and verify its possible symmetry.

36. $f(x) = x^3 - 5x$

37. $f(x) = x^4 - 2x^2 + 1$

38. $f(x) = 2x\sqrt{1 - x^2}$

In Exercises 39–40, the domain of each piecewise function is $(-\infty, \infty)$.

a. *Graph each function.*

b. *Use the graph to determine the function's range.*

39. $f(x) = \begin{cases} 5 & \text{if } x \le -1 \\ -3 & \text{if } x > -1 \end{cases}$

40. $f(x) = \begin{cases} 2x & \text{if } x < 0 \\ -x & \text{if } x \ge 0 \end{cases}$

In Exercises 41–42, find and simplify the difference quotient

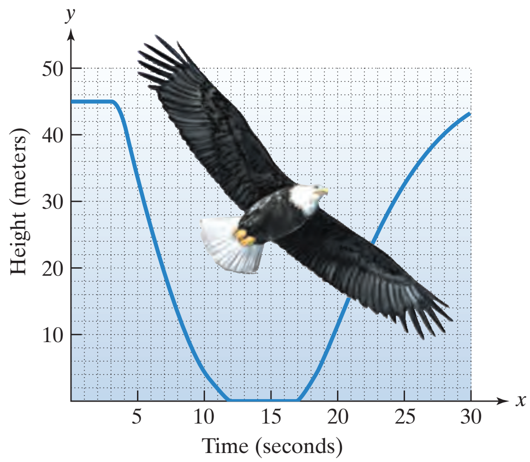
$$\frac{f(x+h) - f(x)}{h}, \quad h \neq 0$$

for the given function.

41. $f(x) = 8x - 11$

42. $f(x) = -2x^2 + x + 10$

43. The graph shows the height, in meters, of an eagle in terms of its time, in seconds, in flight.



a. Is the eagle's height a function of time? Use the graph to explain why or why not.

b. On which interval is the function decreasing? Describe what this means in practical terms.

c. On which intervals is the function constant? What does this mean for each of these intervals?

d. On which interval is the function increasing? What does this mean?

44. A cargo service charges a flat fee of \$5 plus \$1.50 for each pound or fraction of a pound. Graph shipping cost, $C(x)$, in dollars, as a function of weight, x, in pounds, for $0 < x \le 5$.

1.4 and 1.5

In Exercises 45–48, find the slope of the line passing through each pair of points or state that the slope is undefined. Then indicate whether the line through the points rises, falls, is horizontal, or is vertical.

45. $(3, 2)$ and $(5, 1)$

46. $(-1, -2)$ and $(-3, -4)$

47. $\left(-3, \frac{1}{4}\right)$ and $\left(6, \frac{1}{4}\right)$

48. $(-2, 5)$ and $(-2, 10)$

In Exercises 49–52, use the given conditions to write an equation for each line in point-slope form and slope-intercept form.

49. Passing through $(-3, 2)$ with slope -6

50. Passing through $(1, 6)$ and $(-1, 2)$

51. Passing through $(4, -7)$ and parallel to the line whose equation is $3x + y - 9 = 0$

52. Passing through $(-3, 6)$ and perpendicular to the line whose equation is $y = \frac{1}{3}x + 4$

53. Write an equation in general form for the line passing through $(-12, -1)$ and perpendicular to the line whose equation is $6x - y - 4 = 0$.

In Exercises 54–57, give the slope and y-intercept of each line whose equation is given. Then graph the line.

54. $y = \frac{2}{5}x - 1$

55. $f(x) = -4x + 5$

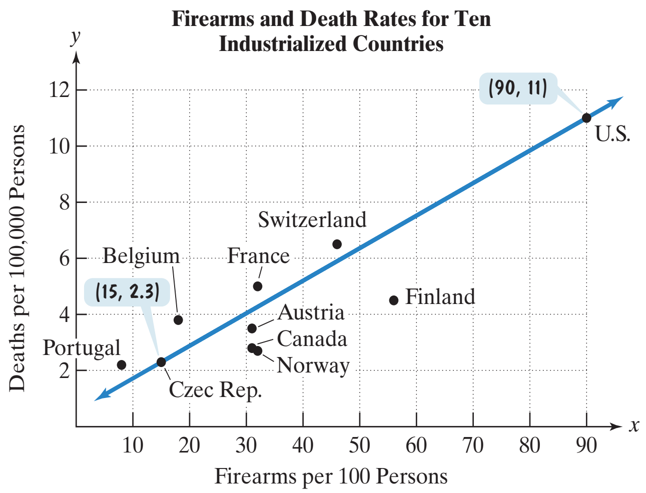
56. $2x + 3y + 6 = 0$

57. $2y - 8 = 0$

58. Graph using intercepts: $2x - 5y - 10 = 0$.

59. Graph: $2x - 10 = 0$.

60. The points in the scatter plot show the number of firearms per 100 persons and the number of deaths per 100,000 persons for industrialized countries with the highest death rates.



Source: International Action Network on Small Arms

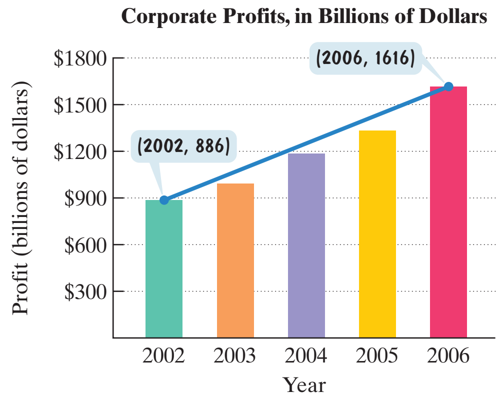
a. Use the two points whose coordinates are shown by the voice balloons to find an equation in point-slope form for the line that models deaths per 100,000 persons, y, as a function of firearms per 100 persons, x.

b. Write the equation in part (a) in slope-intercept form. Use function notation.

c. France has 32 firearms per 100 persons. Use the appropriate point in the scatter plot to estimate that country's deaths per 100,000 persons.

d. Use the function from part (b) to find the number of deaths per 100,000 persons for France. Round to one decimal place. Does the function underestimate or overestimate the deaths per 100,000 persons that you estimated in part (c)? How is this shown by the line in the scatter plot?

61. The bar graph shows the growth in corporate profits, in billions of dollars, from 2002 through 2006.



Source: Bureau of Economic Analysis

Find the slope of the line passing through the two points shown by the voice balloons. Then express the slope as a rate of change with the proper units attached.

62. Find the average rate of change of $f(x) = x^2 - 4x$ from $x_1 = 5$ to $x_2 = 9$.

63. A person standing on the roof of a building throws a ball directly upward. The ball misses the rooftop on its way down and eventually strikes the ground. The function

$$s(t) = -16t^2 + 64t + 80$$

describes the ball's height above the ground, $s(t)$ in feet, t seconds after it was thrown.

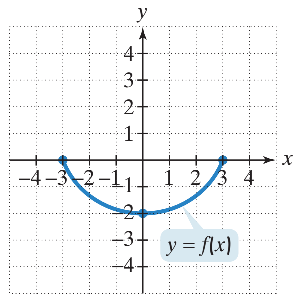
a. Find the ball's average velocity between the time it was thrown and 2 seconds later.

b. Find the ball's average velocity between 2 and 4 seconds after it was thrown.

c. What do the signs in your answers to parts (a) and (b) mean in terms of the direction of the ball's motion?

1.6

In Exercises 64–68, use the graph of $y = f(x)$ to graph each function g.



64. $g(x) = f(x + 2) + 3$

65. $g(x) = \frac{1}{2}f(x - 1)$

66. $g(x) = -f(2x)$

67. $g(x) = 2f\left(\frac{1}{2}x\right)$

68. $g(x) = -f(-x) - 1$

In Exercises 69–72, begin by graphing the standard quadratic function, $f(x) = x^2$. Then use transformations of this graph to graph the given function.

69. $g(x) = x^2 + 2$

70. $h(x) = (x + 2)^2$

71. $r(x) = -(x + 1)^2$

72. $y(x) = \frac{1}{2}(x - 1)^2 + 1$

In Exercises 73–75, begin by graphing the square root function, $f(x) = \sqrt{x}$. Then use transformations of this graph to graph the given function.

73. $g(x) = \sqrt{x + 3}$

74. $h(x) = \sqrt{3 - x}$

75. $r(x) = 2\sqrt{x + 2}$

In Exercises 76–78, begin by graphing the absolute value function, $f(x) = |x|$. Then use transformations of this graph to graph the given function.

76. $g(x) = |x + 2| - 3$

77. $h(x) = -|x - 1| + 1$

78. $r(x) = \frac{1}{2}|x + 2|$

In Exercises 79–81, begin by graphing the standard cubic function, $f(x) = x^3$. Then use transformations of this graph to graph the given function.

79. $g(x) = \frac{1}{2}(x - 1)^3$

80. $h(x) = -(x + 1)^3$

81. $r(x) = \frac{1}{4}x^3 - 1$

In Exercises 82–84, begin by graphing the cube root function, $f(x) = \sqrt[3]{x}$. Then use transformations of this graph to graph the given function.

82. $g(x) = \sqrt[3]{x + 2} - 1$

83. $h(x) = -\sqrt[3]{2x}$

84. $r(x) = -2\sqrt[3]{-x}$

1.7

In Exercises 85–90, find the domain of each function.

85. $f(x) = x^2 + 6x - 3$

86. $g(x) = \dfrac{4}{x - 7}$

87. $h(x) = \sqrt{4 - x}$

88. $f(x) = \dfrac{x}{x^2 + 4x - 21}$

89. $g(x) = \dfrac{\sqrt{x - 2}}{x - 5}$

90. $f(x) = \sqrt{x - 1} + \sqrt{x + 5}$

In Exercises 91–93, find $f + g, f - g, fg$, and $\frac{f}{g}$. Determine the domain for each function.

91. $f(x) = 3x - 1, \quad g(x) = x - 5$

92. $f(x) = x^2 + x + 1, \quad g(x) = x^2 - 1$

93. $f(x) = \sqrt{x + 7}, \quad g(x) = \sqrt{x - 2}$

In Exercises 94–95, find **a.** $(f \circ g)(x)$; **b.** $(g \circ f)(x)$; **c.** $(f \circ g)(3)$.

94. $f(x) = x^2 + 3, \quad g(x) = 4x - 1$

95. $f(x) = \sqrt{x}, \quad g(x) = x + 1$

In Exercises 96–97, find **a.** $(f \circ g)(x)$; **b.** *the domain of* $(f \circ g)$.

96. $f(x) = \dfrac{x + 1}{x - 2}, \quad g(x) = \dfrac{1}{x}$

97. $f(x) = \sqrt{x - 1}, \quad g(x) = x + 3$

In Exercises 98–99, express the given function h as a composition of two functions f and g so that $h(x) = (f \circ g)(x)$.

98. $h(x) = (x^2 + 2x - 1)^4$

99. $h(x) = \sqrt[3]{7x + 4}$

1.8

In Exercises 100–101, find $f(g(x))$ and $g(f(x))$ and determine whether each pair of functions f and g are inverses of each other.

100. $f(x) = \frac{3}{5}x + \frac{1}{2}$ and $g(x) = \frac{5}{3}x - 2$

101. $f(x) = 2 - 5x$ and $g(x) = \dfrac{2 - x}{5}$

The functions in Exercises 102–104 are all one-to-one. For each function,

a. *Find an equation for $f^{-1}(x)$, the inverse function.*

b. *Verify that your equation is correct by showing that $f(f^{-1}(x)) = x$ and $f^{-1}(f(x)) = x$.*

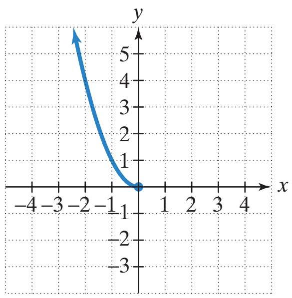
102. $f(x) = 4x - 3$

103. $f(x) = 8x^3 + 1$

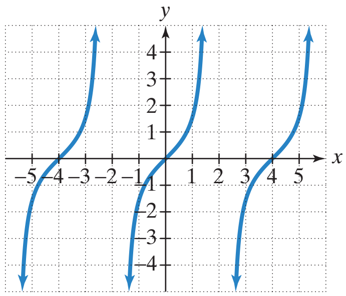
104. $f(x) = \dfrac{2}{x} + 5$

Which graphs in Exercises 105–108 represent functions that have inverse functions?

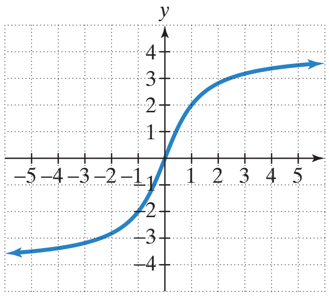
105.



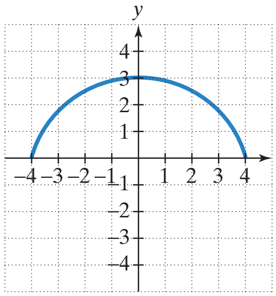
106.



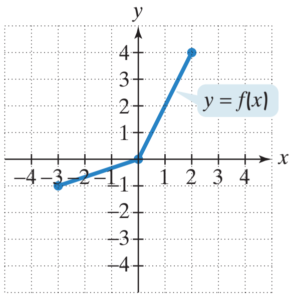
107.



108.



109. Use the graph of f in the figure shown to draw the graph of its inverse function.



In Exercises 110–111, find an equation for $f^{-1}(x)$. Then graph f and f^{-1} in the same rectangular coordinate system.

110. $f(x) = 1 - x^2, x \geq 0$ **111.** $f(x) = \sqrt{x} + 1$

1.9

In Exercises 112–113, find the distance between each pair of points. If necessary, round answers to two decimal places.

112. $(-2, 3)$ and $(3, -9)$ **113.** $(-4, 3)$ and $(-2, 5)$

In Exercises 114–115, find the midpoint of each line segment with the given endpoints.

114. $(2, 6)$ and $(-12, 4)$ **115.** $(4, -6)$ and $(-15, 2)$

In Exercises 116–117, write the standard form of the equation of the circle with the given center and radius.

116. Center $(0, 0), r = 3$ **117.** Center $(-2, 4), r = 6$

In Exercises 118–120, give the center and radius of each circle and graph its equation. Use the graph to identify the relation's domain and range.

118. $x^2 + y^2 = 1$ **119.** $(x + 2)^2 + (y - 3)^2 = 9$

120. $x^2 + y^2 - 4x + 2y - 4 = 0$

1.10

121. In 2000, the average weekly salary for workers in the United States was \$567. This amount has increased by approximately \$15 per year.

a. Express the average weekly salary for U.S. workers, W, as a function of the number of years after 2000, x.

b. If this trend continues, in which year will the average weekly salary be \$702?

122. You are choosing between two long-distance telephone plans. Plan A has a monthly fee of \$15 with a charge of \$0.05 per minute. Plan B has a monthly fee of \$5 with a charge of \$0.07 per minute.

a. Express the monthly cost for plan A, f, as a function of the number of minutes of long-distance calls in a month, x.

b. Express the monthly cost for plan B, g, as a function of the number of minutes of long-distance calls in a month, x.

c. For how many minutes of long-distance calls will the costs for the two plans be the same?

123. A 400-room hotel can rent every one of its rooms at \$120 per room. For each \$1 increase in rent, two fewer rooms are rented.

a. Express the number of rooms rented, N, as a function of the rent, x.

b. Express the hotel's revenue, R, as a function of the rent, x.

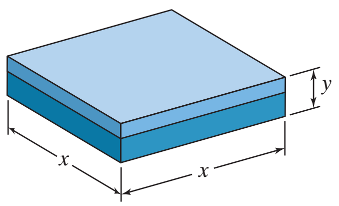
124. An open box is made by cutting identical squares from the corners of a 16-inch by 24-inch piece of cardboard and then turning up the sides.

a. Express the volume of the box, V, as a function of the length of the side of the square cut from each corner, x.

b. Find the domain of V.

125. You have 400 feet of fencing to enclose a rectangular lot and divide it in two by another fence that is parallel to one side of the lot. Express the area of the rectangular lot, A, as a function of the length of the fence that divides the rectangular lot, x.

126. The figure shows a box with a square base and a square top. The box is to have a volume of 8 cubic feet. Express the surface area of the box, A, as a function of the length of a side of its square base, x.



127. You inherit \$10,000 with the stipulation that for the first year the money must be placed in two investments expected to earn 8% and 12% annual interest. Express the expected interest from both investments, I, as a function of the amount of money invested at 8%, x.

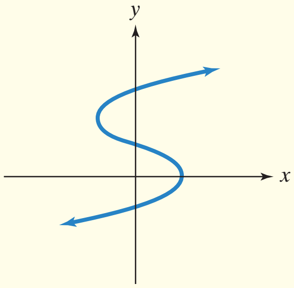
Chapter 1 Test

1. List by letter all relations that are not functions.

a. $\{(7, 5), (8, 5), (9, 5)\}$

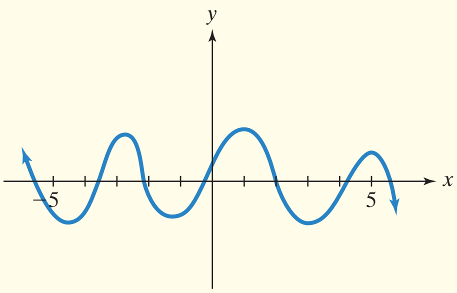
b. $\{(5, 7), (5, 8), (5, 9)\}$

c.

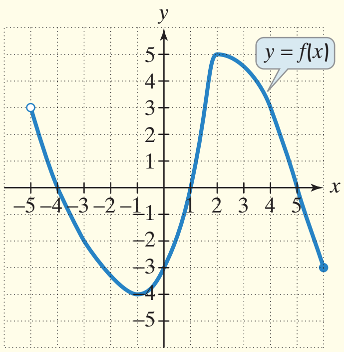


d. $x^2 + y^2 = 100$

e.



2. Use the graph of $y = f(x)$ to solve this exercise.



a. What is $f(4) - f(-3)$?

b. What is the domain of f?

c. What is the range of f?

d. On which interval or intervals is f increasing?

e. On which interval or intervals is f decreasing?

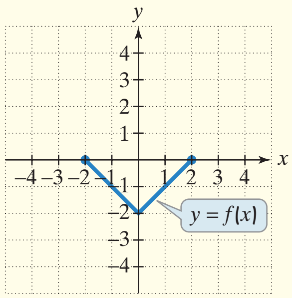
f. For what number does f have a relative maximum? What is the relative maximum?

g. For what number does f have a relative minimum? What is the relative minimum?

h. What are the x-intercepts?

i. What is the y-intercept?

3. Use the graph of $y = f(x)$ to solve this exercise.



a. What are the zeros of f?

b. Find the value(s) of x for which $f(x) = -1$.

c. Find the value(s) of x for which $f(x) = -2$.

d. Is f even, odd, or neither?

e. Does f have an inverse function?

f. Is $f(0)$ a relative maximum, a relative minimum, or neither?

g. Graph $g(x) = f(x + 1) - 1$.

h. Graph $h(x) = \frac{1}{2}f\left(\frac{1}{2}x\right)$.

i. Graph $r(x) = -f(-x) + 1$.

j. Find the average rate of change of f from $x_1 = -2$ to $x_2 = 1$.

In Exercises 4–15, graph each equation in a rectangular coordinate system. If two functions are indicated, graph both in the same system. Then use your graphs to identify each relation's domain and range.

4. $x + y = 4$

5. $x^2 + y^2 = 4$

6. $f(x) = 4$

7. $f(x) = -\frac{1}{3}x + 2$

8. $(x + 2)^2 + (y - 1)^2 = 9$

9. $f(x) = \begin{cases} 2 & \text{if } x \le 0 \\ -1 & \text{if } x > 0 \end{cases}$

10. $x^2 + y^2 + 4x - 6y - 3 = 0$

11. $f(x) = |x|$ and $g(x) = \frac{1}{2}|x + 1| - 2$

12. $f(x) = x^2$ and $g(x) = -(x - 1)^2 + 4$

13. $f(x) = 2x - 4$ and f^{-1}

14. $f(x) = x^3 - 1$ and f^{-1}

15. $f(x) = x^2 - 1, x \ge 0$, and f^{-1}

In Exercises 16–23, let $f(x) = x^2 - x - 4$ and $g(x) = 2x - 6$.

16. Find $f(x - 1)$.

17. Find $\dfrac{f(x + h) - f(x)}{h}$.

18. Find $(g - f)(x)$.

19. Find $\left(\dfrac{f}{g}\right)(x)$ and its domain.

20. Find $(f \circ g)(x)$.

21. Find $(g \circ f)(x)$.

22. Find $g(f(-1))$.

23. Find $f(-x)$. Is f even, odd, or neither?

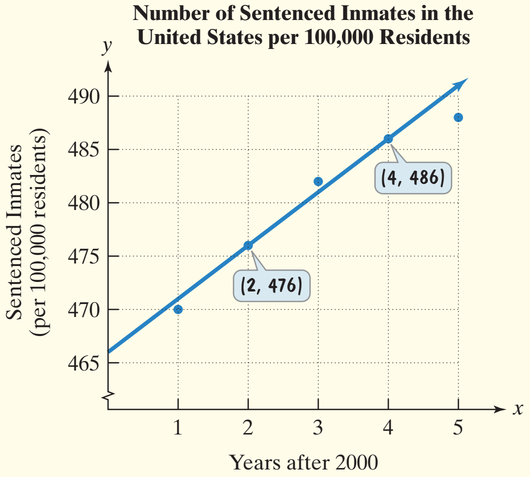
In Exercises 24–25, use the given conditions to write an equation for each line in point-slope form and slope-intercept form.

24. Passing through $(2, 1)$ and $(-1, -8)$

25. Passing through $(-4, 6)$ and perpendicular to the line whose equation is $y = -\frac{1}{4}x + 5$

26. Write an equation in general form for the line passing through $(-7, -10)$ and parallel to the line whose equation is $4x + 2y - 5 = 0$.

27. The scatter plot shows the number of sentenced inmates in the United States per 100,000 residents from 2001 through 2005. Also shown is a line that passes through or near the data points.



Source: U.S. Justice Department

a. Use the two points whose coordinates are shown by the voice balloons to find the point-slope form of the equation of the line that models the number of inmates per 100,000 residents, y, x years after 2000.

b. Write the equation from part (a) in slope-intercept form. Use function notation.

c. Use the linear function to predict the number of sentenced inmates in the United States per 100,000 residents in 2010.

28. Find the average rate of change of $f(x) = 3x^2 - 5$ from $x_1 = 6$ to $x_2 = 10$.

29. If $g(x) = \begin{cases} \sqrt{x-3} & \text{if } x \geq 3 \\ 3 - x & \text{if } x < 3 \end{cases}$, find $g(-1)$ and $g(7)$.

In Exercises 30–31, find the domain of each function.

30. $f(x) = \dfrac{3}{x+5} + \dfrac{7}{x-1}$

31. $f(x) = 3\sqrt{x+5} + 7\sqrt{x-1}$

32. If $f(x) = \dfrac{7}{x-4}$ and $g(x) = \dfrac{2}{x}$, find $(f \circ g)(x)$ and the domain of $f \circ g$.

33. Express $h(x) = (2x + 3)^7$ as a composition of two functions f and g so that $h(x) = (f \circ g)(x)$.

34. Find the length and the midpoint of the line segment whose endpoints are $(2, -2)$ and $(5, 2)$.

35. In 1980, the winning time for women in the Olympic 500-meter speed skating event was 41.78 seconds. The average rate of decrease in the winning time has been about 0.19 second per year.

a. Express the winning time, T, in this event as a function of the number of years after 1980, x.

b. According to the function, when will the winning time be 35.7 seconds?

36. The annual yield per walnut tree is fairly constant at 50 pounds per tree when the number of trees per acre is 30 or fewer. For each additional tree over 30, the annual yield per tree for all trees on the acre decreases by 1.5 pounds due to overcrowding.

a. Express the yield per tree, Y, in pounds, as a function of the number of walnut trees per acre, x.

b. Express the total yield for an acre, T, in pounds, as a function of the number of walnut trees per acre, x.

37. You have 600 yards of fencing to enclose a rectangular field. Express the area of the field, A, as a function of one of its dimensions, x.

38. A closed rectangular box with a square base has a volume of 8000 cubic centimeters. Express the surface area of the box, A, as a function of the length of a side of its square base, x.