Section 1.3

More on Functions and Their Graphs

Objectives

1. Identify intervals on which a function increases, decreases, or is constant.
2. Use graphs to locate relative maxima or minima.
3. Identify even or odd functions and recognize their symmetries.
4. Understand and use piecewise functions.
5. Find and simplify a function's difference quotient.

"For a while we pondered whether to take a vacation or get a divorce. We decided that a trip to Bermuda is over in two weeks, but a divorce is something you always have."
—*Woody Allen*

The *Comedy Thesaurus* (Quirk Books, 2005) contains 23 jokes on the subject of divorce. This section opens with a graph on the subject, intended less for hilarious observations and more for an informal understanding of how we describe graphs of functions. The graph in **Figure 1.27** shows the percent distribution of divorces in the United States by number of years of marriage.

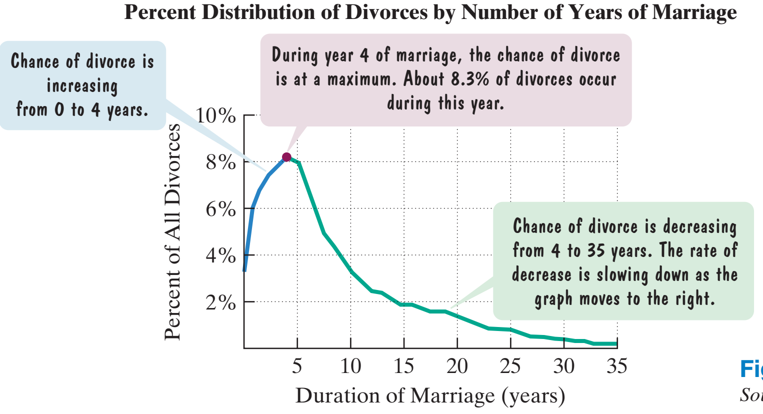


Figure 1.27
Source: Divorce Center

You are probably familiar with the words and phrases used to describe the graph in **Figure 1.27**:

increasing   decreasing   maximum   slowing rate of decrease

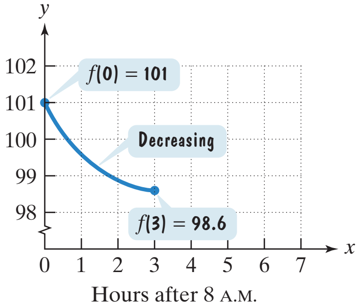
In this section, you will enhance your intuitive understanding of ways of describing graphs by viewing these descriptions from the perspective of functions.

1 Identify intervals on which a function increases, decreases, or is constant.

Increasing and Decreasing Functions

Too late for that flu shot now! It's only 8 A.M. and you're feeling lousy. Your temperature is 101°F. Fascinated by the way that algebra models the world (your author is projecting a bit here), you decide to construct graphs showing your body temperature as a function of the time of day. You decide to let x represent the number of hours after 8 A.M. and $f(x)$ your temperature at time x.

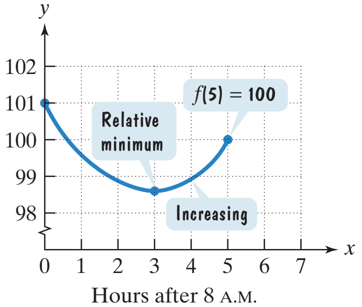
At 8 A.M. your temperature is 101°F and you are not feeling well. However, your temperature starts to decrease. It reaches normal (98.6°F) by 11 A.M. Feeling energized, you construct the graph shown on the right, indicating decreasing temperature for $\{x|0 < x < 3\}$, or on the interval $(0, 3)$.



Temperature decreases on $(0, 3)$, reaching 98.6° by 11 A.M.

Did creating that first graph drain you of your energy? Your temperature starts to rise after 11 A.M. By 1 P.M., 5 hours after 8 A.M., your temperature reaches 100°F. However, you keep plotting points on your graph. At the right, we can see that your temperature increases for $\{x|3 < x < 5\}$, or on the interval $(3, 5)$.

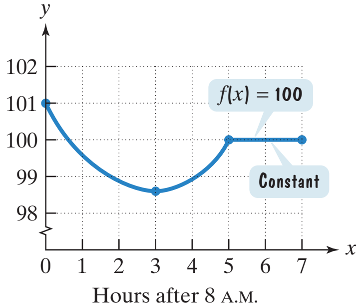
The graph of f is decreasing to the left of $x = 3$ and increasing to the right of $x = 3$. Thus, your temperature 3 hours after 8 A.M. was at its lowest point. Your relative minimum temperature was 98.6°.



Temperature increases on $(3, 5)$.

By 3 P.M., your temperature is no worse than it was at 1 P.M.: It is still 100°F. (Of course, it's no better, either.) Your temperature remained the same, or constant, for $\{x|5 < x < 7\}$, or on the interval $(5, 7)$.

The time-temperature flu scenario illustrates that a function f is increasing when its graph rises from left to right, decreasing when its graph falls from left to right, and remains constant when it neither rises nor falls. Let's now provide a more precise algebraic description for these intuitive concepts.

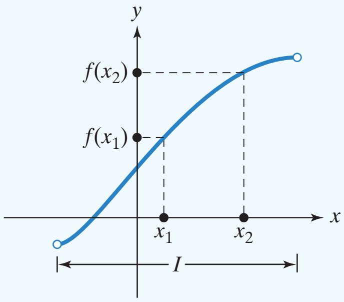


Temperature remains constant at 100° on $(5, 7)$.

Increasing, Decreasing, and Constant Functions

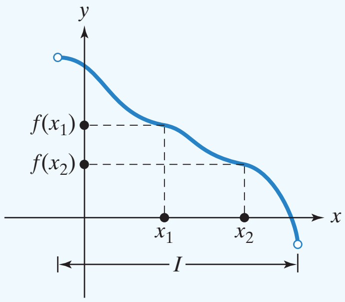
1. A function is **increasing** on an open interval, I, if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.
2. A function is **decreasing** on an open interval, I, if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$ for any x_1 and x_2 in the interval.
3. A function is **constant** on an open interval, I, if $f(x_1) = f(x_2)$ for any x_1 and x_2 in the interval.

Increasing



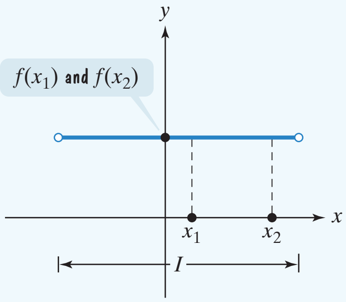
(1) For $x_1 < x_2$ in I, $f(x_1) < f(x_2)$; f is increasing on I.

Decreasing



(2) For $x_1 < x_2$ in I, $f(x_1) > f(x_2)$; f is decreasing on I.

Constant



(3) For x_1 and x_2 in I, $f(x_1) = f(x_2)$; f is constant on I.

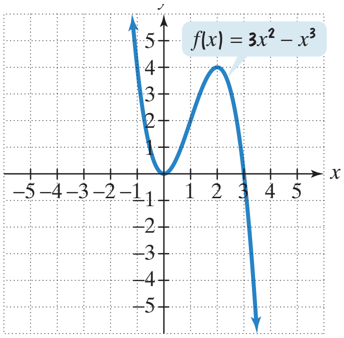
Study Tip

The open intervals describing where functions increase, decrease, or are constant use x-coordinates and not y-coordinates.

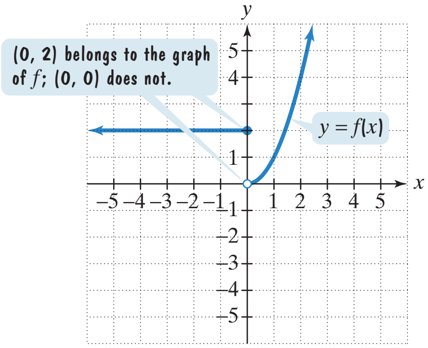
EXAMPLE 1 Intervals on Which a Function Increases, Decreases, or Is Constant

State the intervals on which each given function is increasing, decreasing, or constant.

a.



b.

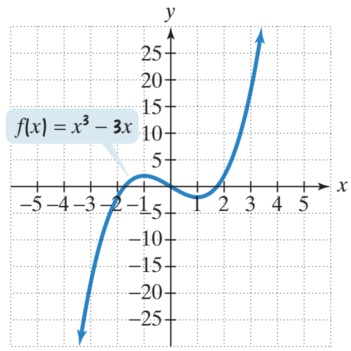


Solution

a. The function is decreasing on the interval $(-\infty, 0)$, increasing on the interval $(0, 2)$, and decreasing on the interval $(2, \infty)$.

b. Although the function's equations are not given, the graph indicates that the function is defined in two pieces. The part of the graph to the left of the y-axis shows that the function is constant on the interval $(-\infty, 0)$. The part to the right of the y-axis shows that the function is increasing on the interval $(0, \infty)$.

Check Point 1 State the intervals on which the given function is increasing, decreasing, or constant.



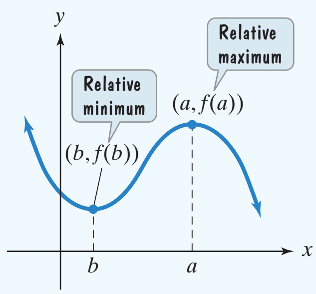
2 Use graphs to locate relative maxima or minima.

Relative Maxima and Relative Minima

The points at which a function changes its increasing or decreasing behavior can be used to find the *relative maximum* or *relative minimum* values of the function.

Definitions of Relative Maximum and Relative Minimum

1. A function value $f(a)$ is a **relative maximum** of f if there exists an open interval containing a such that $f(a) > f(x)$ for all $x \neq a$ in the open interval.
2. A function value $f(b)$ is a **relative minimum** of f if there exists an open interval containing b such that $f(b) < f(x)$ for all $x \neq b$ in the open interval.



Study Tip

The word *local* is sometimes used instead of *relative* when describing maxima or minima. If f has a relative, or local, maximum at a, $f(a)$ is greater than all other values of f near a. If f has a relative, or local, minimum at b, $f(b)$ is less than all other values of f near b.

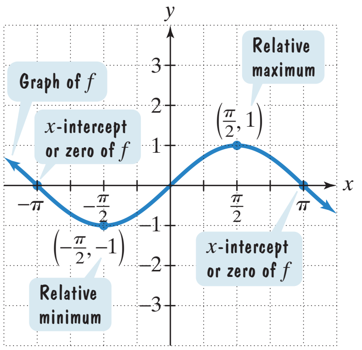


Figure 1.28 Using a graph to locate where a function has a relative maximum or minimum

If the graph of a function is given, we can often visually locate the number(s) at which the function has a relative maximum or a relative minimum. For example, the graph of f in **Figure 1.28** shows that

- f has a relative maximum at $\frac{\pi}{2}$.

  The relative maximum is $f\left(\frac{\pi}{2}\right) = 1$.

- f has a relative minimum at $-\frac{\pi}{2}$.

  The relative minimum is $f\left(-\frac{\pi}{2}\right) = -1$.

Notice that f does not have a relative maximum or minimum at $-\pi$ and π, the x-intercepts, or zeros, of the function.

3 Identify even or odd functions and recognize their symmetries.

Even and Odd Functions and Symmetry

Figure 1.29 To most people, an attractive face is one in which each half is an almost perfect mirror image of the other half.

Is beauty in the eye of the beholder? Or are there certain objects (or people) that are so well balanced and proportioned that they are universally pleasing to the eye? What constitutes an attractive human face? In **Figure 1.29**, we've drawn lines between paired features and marked the midpoints. Notice how the features line up almost perfectly. Each half of the face is a mirror image of the other half through the white vertical line.

Did you know that graphs of some equations exhibit exactly the kind of symmetry shown by the attractive face in **Figure 1.29**? The word *symmetry* comes from the Greek *symmetria*, meaning "the same measure." We can identify graphs with symmetry by looking at a function's equation and determining if the function is *even* or *odd*.

Definitions of Even and Odd Functions

The function f is an **even function** if

$$f(-x) = f(x) \quad \text{for all } x \text{ in the domain of } f.$$

The right side of the equation of an even function does not change if x is replaced with $-x$.

The function f is an **odd function** if

$$f(-x) = -f(x) \quad \text{for all } x \text{ in the domain of } f.$$

Every term on the right side of the equation of an odd function changes its sign if x is replaced with $-x$.

EXAMPLE 2 Identifying Even or Odd Functions

Determine whether each of the following functions is even, odd, or neither:

a. $f(x) = x^3 - 6x$ **b.** $g(x) = x^4 - 2x^2$ **c.** $h(x) = x^2 + 2x + 1$.

Solution In each case, replace x with $-x$ and simplify. If the right side of the equation stays the same, the function is even. If every term on the right side changes sign, the function is odd.

a. We use the given function's equation, $f(x) = x^3 - 6x$, to find $f(-x)$.

Use $f(x) = x^3 - 6x$.

Replace x with $-x$.

$$f(-x) = (-x)^3 - 6(-x) = (-x)(-x)(-x) - 6(-x) = -x^3 + 6x$$

There are two terms on the right side of the given equation, $f(x) = x^3 - 6x$, and each term changed its sign when we replaced x with $-x$. Because $f(-x) = -f(x)$, f is an odd function.

b. We use the given function's equation, $g(x) = x^4 - 2x^2$, to find $g(-x)$.

Use $g(x) = x^4 - 2x^2$.

Replace x with $-x$.

$$g(-x) = (-x)^4 - 2(-x)^2 = (-x)(-x)(-x)(-x) - 2(-x)(-x)$$
$$= x^4 - 2x^2$$

The right side of the equation of the given function, $g(x) = x^4 - 2x^2$, did not change when we replaced x with $-x$. Because $g(-x) = g(x)$, g is an even function.

c. We use the given function's equation, $h(x) = x^2 + 2x + 1$, to find $h(-x)$.

Use $h(x) = x^2 + 2x + 1$.

Replace x with $-x$.

$$h(-x) = (-x)^2 + 2(-x) + 1 = x^2 - 2x + 1$$

The right side of the equation of the given function, $h(x) = x^2 + 2x + 1$, changed when we replaced x with $-x$. Thus, $h(-x) \neq h(x)$, so h is not an even function. The sign of *each* of the three terms in the equation for $h(x)$ did not change when we replaced x with $-x$. Only the second term changed signs. Thus, $h(-x) \neq -h(x)$, so h is not an odd function. We conclude that h is neither an even nor an odd function.

Check Point 2 Determine whether each of the following functions is even, odd, or neither:

a. $f(x) = x^2 + 6$ **b.** $g(x) = 7x^3 - x$ **c.** $h(x) = x^5 + 1$.

Now, let's see what even and odd functions tell us about a function's graph. Begin with the even function $f(x) = x^2 - 4$, shown in **Figure 1.30**. The function is even because

$$f(-x) = (-x)^2 - 4 = x^2 - 4 = f(x).$$

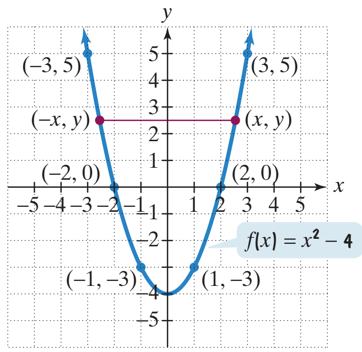


Figure 1.30 y-axis symmetry with $f(-x) = f(x)$

Examine the pairs of points shown, such as (3, 5) and (−3, 5). Notice that we obtain the same y-coordinate whenever we evaluate the function at a value of x and the value of its opposite, $-x$. Like the attractive face, each half of the graph is a mirror image of the other half through the y-axis. If we were to fold the paper along the y-axis, the two halves of the graph would coincide. This is what it means for the graph to be *symmetric with respect to the y-axis*. A graph is **symmetric with respect to the *y*-axis** if, for every point (x, y) on the graph, the point $(-x, y)$ is also on the graph. All even functions have graphs with this kind of symmetry.

Even Functions and *y*-Axis Symmetry

The graph of an even function in which $f(-x) = f(x)$ is symmetric with respect to the y-axis.

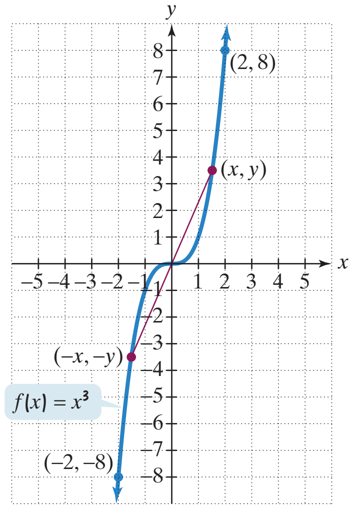


Figure 1.31 Origin symmetry with $f(-x) = -f(x)$

Now, consider the graph of the function $f(x) = x^3$, shown in **Figure 1.31**. The function is odd because

$$f(-x) = (-x)^3 = (-x)(-x)(-x) = -x^3 = -f(x).$$

Although the graph in **Figure 1.31** is not symmetric with respect to the y-axis, it is symmetric in another way. Look at the pairs of points, such as (2, 8) and (−2, −8). For each point (x, y) on the graph, the point $(-x, -y)$ is also on the graph. The points (2, 8) and (−2, −8) are reflections of one another through the origin. This means that the origin is the midpoint of the line segment connecting the points.

A graph is **symmetric with respect to the origin** if, for every point (x, y) on the graph, the point $(-x, -y)$ is also on the graph. Observe that the first- and third-quadrant portions of $f(x) = x^3$ are reflections of one another with respect to the origin. Notice that $f(x)$ and $f(-x)$ have opposite signs, so that $f(-x) = -f(x)$. All odd functions have graphs with origin symmetry.

Odd Functions and Origin Symmetry

The graph of an odd function in which $f(-x) = -f(x)$ is symmetric with respect to the origin.

4 Understand and use piecewise functions.

Piecewise Functions

A cellular phone company offers the following plan:

- $20 per month buys 60 minutes.
- Additional time costs $0.40 per minute.

We can represent this plan mathematically by writing the total monthly cost, C, as a function of the number of calling minutes, t.

$$C(t) = \begin{cases} 20 & \text{if } 0 \le t \le 60 \\ 20 + 0.40(t - 60) & \text{if } t > 60 \end{cases}$$

The cost is $20 for up to and including 60 calling minutes.

The cost is $20 plus $0.40 per minute for additional time for more than 60 calling minutes.

$20 for first 60 minutes | $0.40 per minute | times the number of calling minutes exceeding 60

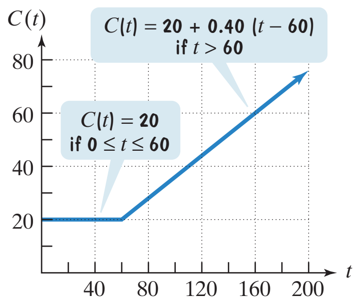


Figure 1.32

A function that is defined by two (or more) equations over a specified domain is called a **piecewise function**. Many cellular phone plans can be represented with piecewise functions. The graph of the piecewise function described above is shown in **Figure 1.32**.

EXAMPLE 3 Evaluating a Piecewise Function

Use the function that describes the cellular phone plan

$$C(t) = \begin{cases} 20 & \text{if } 0 \le t \le 60 \\ 20 + 0.40(t - 60) & \text{if } t > 60 \end{cases}$$

to find and interpret each of the following:

a. $C(30)$ **b.** $C(100)$.

Solution

a. To find $C(30)$, we let $t = 30$. Because 30 lies between 0 and 60, we use the first line of the piecewise function.

$C(t) = 20$ This is the function's equation for $0 \le t \le 60$.

$C(30) = 20$ Replace t with 30. Regardless of this function's input, the constant output is 20.

This means that with 30 calling minutes, the monthly cost is $20. This can be visually represented by the point (30, 20) on the first piece of the graph in **Figure 1.32**.

$$C(t) = \begin{cases} 20 & \text{if } 0 \le t \le 60 \\ 20 + 0.40(t - 60) & \text{if } t > 60 \end{cases}$$

The given piecewise function (repeated)

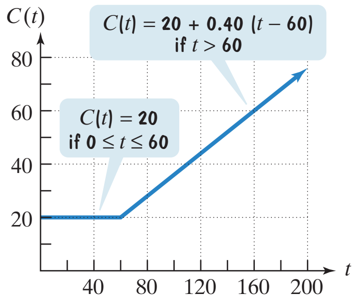


Figure 1.32 (repeated)

b. To find $C(100)$, we let $t = 100$. Because 100 is greater than 60, we use the second line of the piecewise function.

$$\begin{aligned} C(t) &= 20 + 0.40(t - 60) && \text{This is the function's equation for } t > 60. \\ C(100) &= 20 + 0.40(100 - 60) && \text{Replace } t \text{ with } 100. \\ &= 20 + 0.40(40) && \text{Subtract within parentheses: } 100 - 60 = 40. \\ &= 20 + 16 && \text{Multiply: } 0.40(40) = 16. \\ &= 36 && \text{Add: } 20 + 16 = 36. \end{aligned}$$

Thus, $C(100) = 36$. This means that with 100 calling minutes, the monthly cost is \$36. This can be visually represented by the point (100, 36) on the second piece of the graph in **Figure 1.32**.

Check Point 3 Use the function in Example 3 to find and interpret each of the following:

a. $C(40)$ **b.** $C(80)$.

Identify your solutions by points on the graph in **Figure 1.32**.

EXAMPLE 4 Graphing a Piecewise Function

Graph the piecewise function defined by

$$f(x) = \begin{cases} x + 2 & \text{if } \quad x \le 1 \\ 4 & \text{if } \quad x > 1. \end{cases}$$

Solution We graph f in two parts, using a partial table of coordinates to create each piece. The tables of coordinates and the completed graph are shown in **Figure 1.33**.

Graph $f(x) = x + 2$ for $x \le 1$.
Domain = $(-\infty, 1]$

x $(x \le 1)$	$f(x) = x + 2$	$(x, f(x))$
1	$f(1) = 1 + 2 = 3$	(1, 3)
0	$f(0) = 0 + 2 = 2$	(0, 2)
−1	$f(-1) = -1 + 2 = 1$	(−1, 1)
−2	$f(-2) = -2 + 2 = 0$	(−2, 0)
−3	$f(-3) = -3 + 2 = -1$	(−3, −1)
−4	$f(-4) = -4 + 2 = -2$	(−4, −2)

Graph $f(x) = 4$ for $x > 1$.
Domain = $(1, \infty)$

x $(x > 1)$	$f(x) = 4$	$(x, f(x))$
1.5	$f(1.5) = 4$	(1.5, 4)
2	$f(2) = 4$	(2, 4)
3	$f(3) = 4$	(3, 4)
4	$f(4) = 4$	(4, 4)
5	$f(5) = 4$	(5, 4)

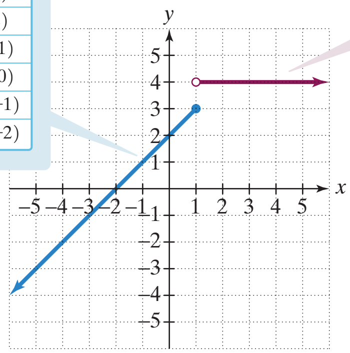


Figure 1.33 Graphing a piecewise function

We can use the graph of the piecewise function in **Figure 1.33** to find the range of f. The range of the piece on the left is $\{y | y \le 3\}$. The range of the horizontal piece on the right is $\{y | y = 4\}$. Thus, the range of f is

$$\{y | y \le 3\} \cup \{y | y = 4\}, \text{ or } (-\infty, 3] \cup \{4\}.$$

Check Point 4 Graph the piecewise function defined by

$$f(x) = \begin{cases} 3 & \text{if} \quad x \le -1 \\ x - 2 & \text{if} \quad x > -1. \end{cases}$$

Some piecewise functions are called **step functions** because their graphs form discontinuous steps. One such function is called the **greatest integer function**, symbolized by $\text{int}(x)$ or $[\![x]\!]$, where

$$\text{int}(x) = \text{the greatest integer that is less than or equal to } x.$$

For example,

$$\text{int}(1) = 1, \quad \text{int}(1.3) = 1, \quad \text{int}(1.5) = 1, \quad \text{int}(1.9) = 1.$$

1 is the greatest integer that is less than or equal to 1, 1.3, 1.5, and 1.9.

Here are some additional examples:

$$\text{int}(2) = 2, \quad \text{int}(2.3) = 2, \quad \text{int}(2.5) = 2, \quad \text{int}(2.9) = 2.$$

2 is the greatest integer that is less than or equal to 2, 2.3, 2.5, and 2.9.

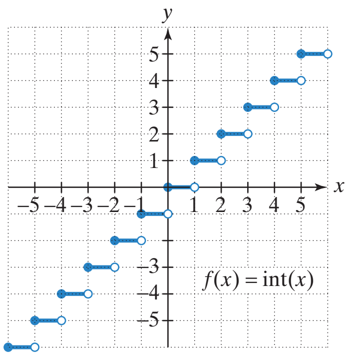


Figure 1.34 The graph of the greatest integer function

Notice how we jumped from 1 to 2 in the function values for $\text{int}(x)$. In particular,

$$\text{If } 1 \le x < 2, \quad \text{then} \quad \text{int}(x) = 1.$$
$$\text{If } 2 \le x < 3, \quad \text{then} \quad \text{int}(x) = 2.$$

The graph of $f(x) = \text{int}(x)$ is shown in **Figure 1.34**. The graph of the greatest integer function jumps vertically one unit at each integer. However, the graph is constant between each pair of consecutive integers. The rightmost horizontal step shown in the graph illustrates that

$$\text{If } 5 \le x < 6, \quad \text{then} \quad \text{int}(x) = 5.$$

In general,

$$\text{If } n \le x < n + 1, \text{ where } n \text{ is an integer}, \quad \text{then} \quad \text{int}(x) = n.$$

5 Find and simplify a function's difference quotient.

Functions and Difference Quotients

In the next section, we will be studying the average rate of change of a function. A ratio, called the *difference quotient*, plays an important role in understanding the rate at which functions change.

Definition of the Difference Quotient of a Function

The expression

$$\frac{f(x + h) - f(x)}{h}$$

for $h \neq 0$ is called the **difference quotient** of the function f.

EXAMPLE 5 Evaluating and Simplifying a Difference Quotient

If $f(x) = 2x^2 - x + 3$, find and simplify each expression:

a. $f(x + h)$ **b.** $\dfrac{f(x + h) - f(x)}{h}, h \neq 0.$

Solution

a. We find $f(x + h)$ by replacing x with $x + h$ each time that x appears in the equation.

$$f(x) = 2x^2 - x + 3$$

Replace x with $x + h$. Replace x with $x + h$. Replace x with $x + h$. Copy the 3. There is no x in this term.

$$\begin{aligned} f(x+h) &= 2(x+h)^2 - (x+h) + 3 \\ &= 2(x^2 + 2xh + h^2) - x - h + 3 \\ &= 2x^2 + 4xh + 2h^2 - x - h + 3 \end{aligned}$$

b. Using our result from part (a), we obtain the following:

This is $f(x + h)$ from part (a). This is $f(x)$ from the given equation.

$$\frac{f(x+h) - f(x)}{h} = \frac{\boxed{2x^2 + 4xh + 2h^2 - x - h + 3} - (2x^2 - x + 3)}{h}$$

$$= \frac{2x^2 + 4xh + 2h^2 - x - h + 3 - 2x^2 + x - 3}{h}$$ Remove parentheses and change the sign of each term in the parentheses.

$$= \frac{(2x^2 - 2x^2) + (-x + x) + (3 - 3) + 4xh + 2h^2 - h}{h}$$ Group like terms.

$$= \frac{4xh + 2h^2 - 1h}{h}$$ Simplify.

We wrote $-h$ as $-1h$ to avoid possible errors in the next factoring step.

$$= \frac{h(4x + 2h - 1)}{h}$$ Factor h from the numerator.

$$= 4x + 2h - 1$$ Divide out identical factors of h in the numerator and denominator.

Check Point 5 If $f(x) = -2x^2 + x + 5$, find and simplify each expression:

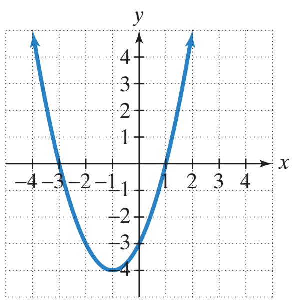
a. $f(x + h)$ **b.** $\dfrac{f(x+h) - f(x)}{h}$, $h \neq 0$.

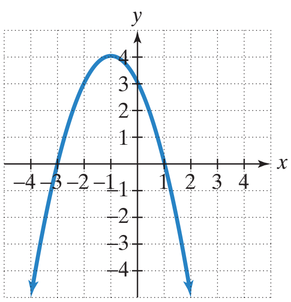
Exercise Set 1.3

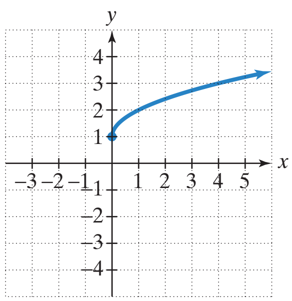
Practice Exercises

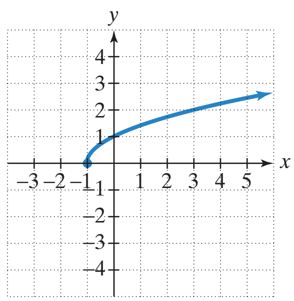
In Exercises 1–12, use the graph to determine

a. *intervals on which the function is increasing, if any.*
b. *intervals on which the function is decreasing, if any.*
c. *intervals on which the function is constant, if any.*

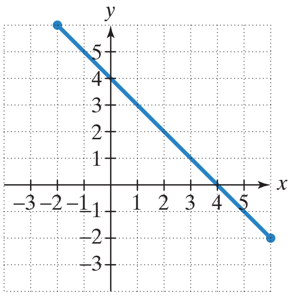
1.


2.


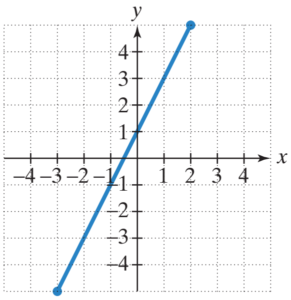
3.


4.


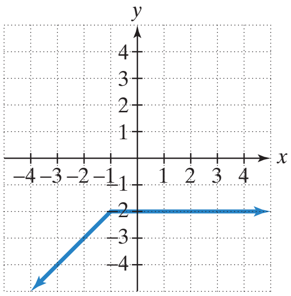
5.



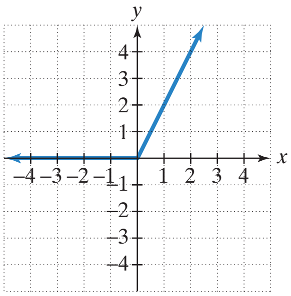
6.



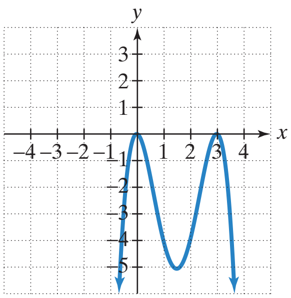
7.



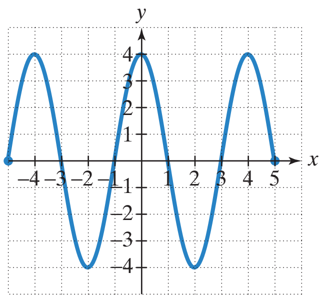
8.



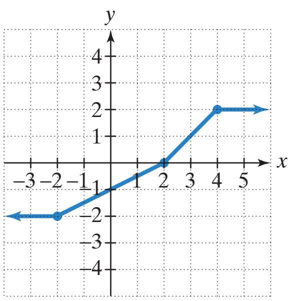
9.



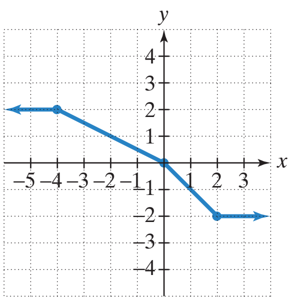
10.



11.



12.

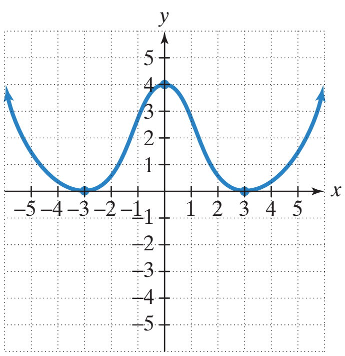


In Exercises 13–16, the graph of a function f is given. Use the graph to find each of the following:

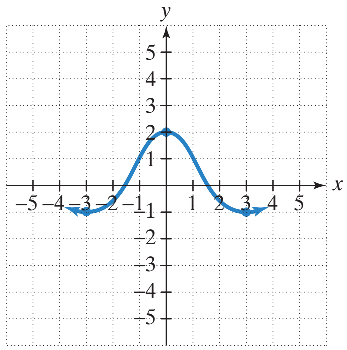
a. *The numbers, if any, at which f has a relative maximum. What are these relative maxima?*

b. *The numbers, if any, at which f has a relative minimum. What are these relative minima?*

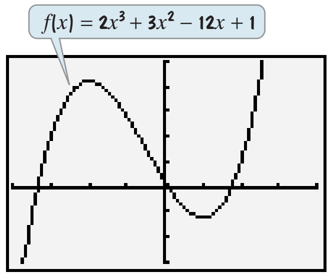
13.



14.

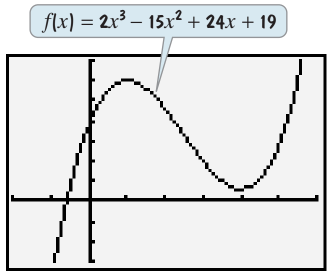


15.



[–4, 4, 1] by [–15, 25, 5]

16.



[–2, 6, 1] by [–15, 35, 5]

In Exercises 17–28, determine whether each function is even, odd, or neither.

17. $f(x) = x^3 + x$

18. $f(x) = x^3 - x$

19. $g(x) = x^2 + x$

20. $g(x) = x^2 - x$

21. $h(x) = x^2 - x^4$

22. $h(x) = 2x^2 + x^4$

23. $f(x) = x^2 - x^4 + 1$

24. $f(x) = 2x^2 + x^4 + 1$

25. $f(x) = \frac{1}{5}x^6 - 3x^2$

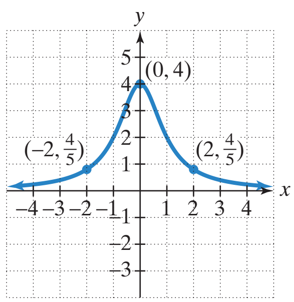
26. $f(x) = 2x^3 - 6x^5$

27. $f(x) = x\sqrt{1 - x^2}$

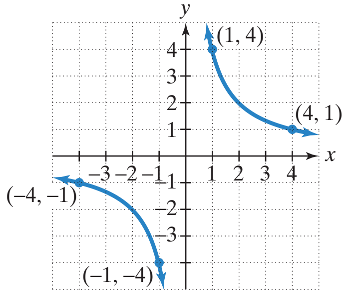
28. $f(x) = x^2\sqrt{1 - x^2}$

In Exercises 29–32, use possible symmetry to determine whether each graph is the graph of an even function, an odd function, or a function that is neither even nor odd.

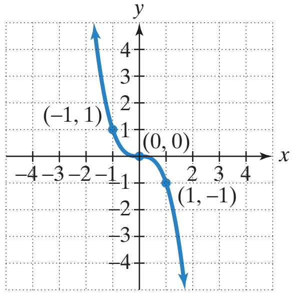
29.



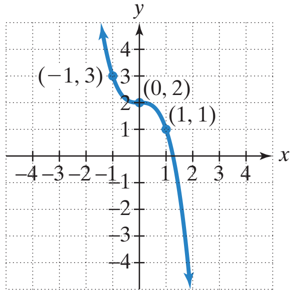
30.



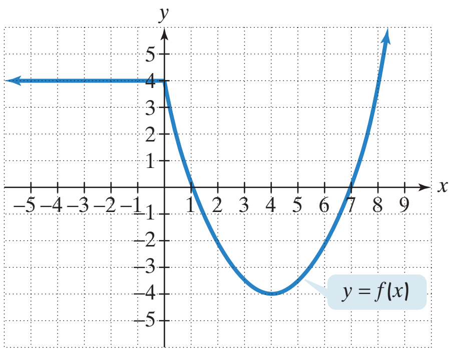
31.



32.



33. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the x-intercepts

d. the y-intercept

e. intervals on which f is increasing

f. intervals on which f is decreasing

g. intervals on which f is constant

h. the number at which f has a relative minimum

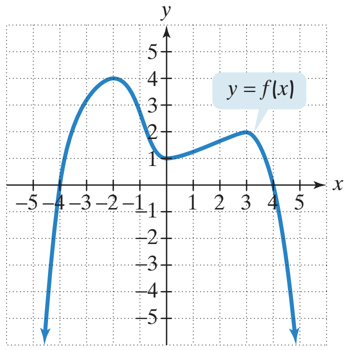
i. the relative minimum of f

j. $f(-3)$

k. the values of x for which $f(x) = -2$

l. Is f even, odd, or neither?

34. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the x-intercepts

d. the y-intercept

e. intervals on which f is increasing

f. intervals on which f is decreasing

g. values of x for which $f(x) \le 0$

h. the numbers at which f has a relative maximum

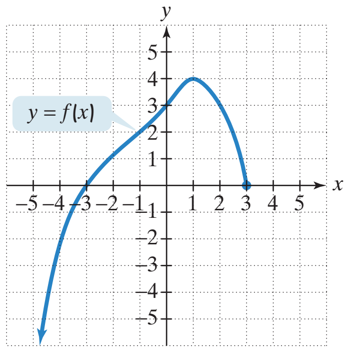
i. the relative maxima of f

j. $f(-2)$

k. the values of x for which $f(x) = 0$

l. Is f even, odd, or neither?

35. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f

b. the range of f

c. the zeros of f

d. $f(0)$

e. intervals on which f is increasing

f. intervals on which f is decreasing

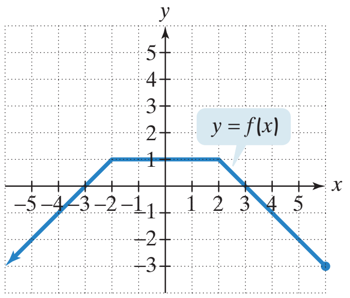
g. values of x for which $f(x) \le 0$

h. any relative maxima and the numbers at which they occur

i. the value of x for which $f(x) = 4$

j. Is $f(-1)$ positive or negative?

36. Use the graph of f to determine each of the following. Where applicable, use interval notation.



a. the domain of f
b. the range of f
c. the zeros of f
d. $f(0)$
e. intervals on which f is increasing
f. intervals on which f is decreasing
g. intervals on which f is constant
h. values of x for which $f(x) > 0$
i. values of x for which $f(x) = -2$
j. Is $f(4)$ positive or negative?
k. Is f even, odd, or neither?
l. Is $f(2)$ a relative maximum?

In Exercises 37–42, evaluate each piecewise function at the given values of the independent variable.

37. $f(x) = \begin{cases} 3x + 5 & \text{if } x < 0 \\ 4x + 7 & \text{if } x \geq 0 \end{cases}$

a. $f(-2)$ **b.** $f(0)$ **c.** $f(3)$

38. $f(x) = \begin{cases} 6x - 1 & \text{if } x < 0 \\ 7x + 3 & \text{if } x \geq 0 \end{cases}$

a. $f(-3)$ **b.** $f(0)$ **c.** $f(4)$

39. $g(x) = \begin{cases} x + 3 & \text{if } x \geq -3 \\ -(x + 3) & \text{if } x < -3 \end{cases}$

a. $g(0)$ **b.** $g(-6)$ **c.** $g(-3)$

40. $g(x) = \begin{cases} x + 5 & \text{if } x \geq -5 \\ -(x + 5) & \text{if } x < -5 \end{cases}$

a. $g(0)$ **b.** $g(-6)$ **c.** $g(-5)$

41. $h(x) = \begin{cases} \dfrac{x^2 - 9}{x - 3} & \text{if } x \neq 3 \\ 6 & \text{if } x = 3 \end{cases}$

a. $h(5)$ **b.** $h(0)$ **c.** $h(3)$

42. $h(x) = \begin{cases} \dfrac{x^2 - 25}{x - 5} & \text{if } x \neq 5 \\ 10 & \text{if } x = 5 \end{cases}$

a. $h(7)$ **b.** $h(0)$ **c.** $h(5)$

In Exercises 43–54, the domain of each piecewise function is $(-\infty, \infty)$.

a. *Graph each function.*

b. *Use your graph to determine the function's range.*

43. $f(x) = \begin{cases} -x & \text{if } x < 0 \\ x & \text{if } x \geq 0 \end{cases}$

44. $f(x) = \begin{cases} x & \text{if } x < 0 \\ -x & \text{if } x \geq 0 \end{cases}$

45. $f(x) = \begin{cases} 2x & \text{if } x \leq 0 \\ 2 & \text{if } x > 0 \end{cases}$

46. $f(x) = \begin{cases} \frac{1}{2}x & \text{if } x \leq 0 \\ 3 & \text{if } x > 0 \end{cases}$

47. $f(x) = \begin{cases} x + 3 & \text{if } x < -2 \\ x - 3 & \text{if } x \geq -2 \end{cases}$

48. $f(x) = \begin{cases} x + 2 & \text{if } x < -3 \\ x - 2 & \text{if } x \geq -3 \end{cases}$

49. $f(x) = \begin{cases} 3 & \text{if } x \leq -1 \\ -3 & \text{if } x > -1 \end{cases}$

50. $f(x) = \begin{cases} 4 & \text{if } x \leq -1 \\ -4 & \text{if } x > -1 \end{cases}$

51. $f(x) = \begin{cases} \frac{1}{2}x^2 & \text{if } x < 1 \\ 2x - 1 & \text{if } x \geq 1 \end{cases}$

52. $f(x) = \begin{cases} -\frac{1}{2}x^2 & \text{if } x < 1 \\ 2x + 1 & \text{if } x \geq 1 \end{cases}$

53. $f(x) = \begin{cases} 0 & \text{if } x < -4 \\ -x & \text{if } -4 \leq x < 0 \\ x^2 & \text{if } x \geq 0 \end{cases}$

54. $f(x) = \begin{cases} 0 & \text{if } x < -3 \\ -x & \text{if } -3 \leq x < 0 \\ x^2 - 1 & \text{if } x \geq 0 \end{cases}$

In Exercises 55–76, find and simplify the difference quotient

$$\frac{f(x + h) - f(x)}{h}, h \neq 0$$

for the given function.

55. $f(x) = 4x$

56. $f(x) = 7x$

57. $f(x) = 3x + 7$

58. $f(x) = 6x + 1$

59. $f(x) = x^2$

60. $f(x) = 2x^2$

61. $f(x) = x^2 - 4x + 3$

62. $f(x) = x^2 - 5x + 8$

63. $f(x) = 2x^2 + x - 1$

64. $f(x) = 3x^2 + x + 5$

65. $f(x) = -x^2 + 2x + 4$

66. $f(x) = -x^2 - 3x + 1$

67. $f(x) = -2x^2 + 5x + 7$

68. $f(x) = -3x^2 + 2x - 1$

69. $f(x) = -2x^2 - x + 3$

70. $f(x) = -3x^2 + x - 1$

71. $f(x) = 6$

72. $f(x) = 7$

73. $f(x) = \frac{1}{x}$

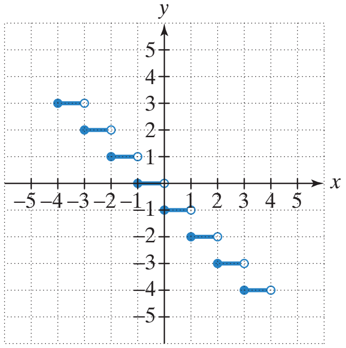
74. $f(x) = \frac{1}{2x}$

75. $f(x) = \sqrt{x}$

76. $f(x) = \sqrt{x - 1}$

Practice Plus

In Exercises 77–78, let f be defined by the following graph:



77. Find

$$\sqrt{f(-1.5) + f(-0.9)} - [f(\pi)]^2 + f(-3) \div f(1) \cdot f(-\pi).$$

78. Find

$$\sqrt{f(-2.5) - f(1.9)} - [f(-\pi)]^2 + f(-3) \div f(1) \cdot f(\pi).$$

A cellular phone company offers the following plans. Also given are the piecewise functions that model these plans. Use this information to solve Exercises 79–80.

Plan A

- *\$30 per month buys 120 minutes.*
- *Additional time costs \$0.30 per minute.*

$$C(t) = \begin{cases} 30 & \text{if } 0 \le t \le 120 \\ 30 + 0.30(t - 120) & \text{if } t > 120 \end{cases}$$

Plan B

- *\$40 per month buys 200 minutes.*
- *Additional time costs \$0.30 per minute.*

$$C(t) = \begin{cases} 40 & \text{if } 0 \le t \le 200 \\ 40 + 0.30(t - 200) & \text{if } t > 200 \end{cases}$$

79. Simplify the algebraic expression in the second line of the piecewise function for plan A. Then use point-plotting to graph the function.

80. Simplify the algebraic expression in the second line of the piecewise function for plan B. Then use point-plotting to graph the function.

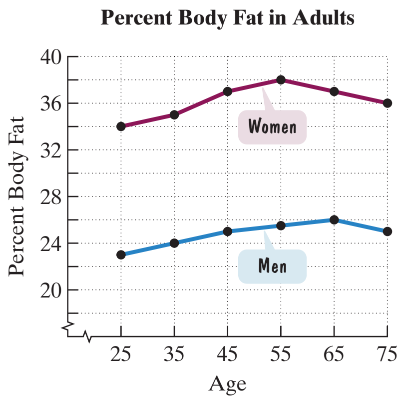
In Exercises 81–82, write a piecewise function that models each cellular phone billing plan. Then graph the function.

81. \$50 per month buys 400 minutes. Additional time costs \$0.30 per minute.

82. \$60 per month buys 450 minutes. Additional time costs \$0.35 per minute.

Application Exercises

With aging, body fat increases and muscle mass declines. The line graphs show the percent body fat in adult women and men as they age from 25 to 75 years. Use the graphs to solve Exercises 83–90.



Source: Thompson et al., *The Science of Nutrition*, Benjamin Cummings, 2008

83. State the intervals on which the graph giving the percent body fat in women is increasing and decreasing.

84. State the intervals on which the graph giving the percent body fat in men is increasing and decreasing.

85. For what age does the percent body fat in women reach a maximum? What is the percent body fat for that age?

86. At what age does the percent body fat in men reach a maximum? What is the percent body fat for that age?

87. Use interval notation to give the domain and the range for the graph of the function for women.

88. Use interval notation to give the domain and the range for the graph of the function for men.

89. The function $p(x) = -0.002x^2 + 0.15x + 22.86$ models percent body fat, $p(x)$, where x is the number of years a person's age exceeds 25. Use the graphs to determine whether this model describes percent body fat in women or in men.

90. The function $p(x) = -0.004x^2 + 0.25x + 33.64$ models percent body fat, $p(x)$, where x is the number of years a person's age exceeds 25. Use the graphs to determine whether this model describes percent body fat in women or in men.

Here is the 2007 Federal Tax Rate Schedule X *that specifies the tax owed by a single taxpayer.*

If Your Taxable Income Is Over	But Not Over	The Tax You Owe Is	Of the Amount Over
\$ 0	\$ 7825	10%	\$ 0
\$ 7825	\$ 31,850	\$ 782.50 + 15%	\$ 7825
\$ 31,850	\$ 77,100	\$ 4386.25 + 25%	\$ 31,850
\$ 77,100	\$160,850	\$ 15,698.75 + 28%	\$ 77,100
\$160,850	\$349,700	\$ 39,148.75 + 33%	\$160,850
\$349,700	—	\$101,469.25 + 35%	\$349,700

The tax table on the previous page can be modeled by a piecewise function, where x represents the taxable income of a single taxpayer and $T(x)$ is the tax owed:

$$T(x) = \begin{cases} 0.10x & \text{if} \quad 0 < x \le 7825 \\ 782.50 + 0.15(x - 7825) & \text{if} \quad 7825 < x \le 31{,}850 \\ 4386.25 + 0.25(x - 31{,}850) & \text{if} \quad 31{,}850 < x \le 77{,}100 \\ 15{,}698.75 + 0.28(x - 77{,}100) & \text{if} \quad 77{,}100 < x \le 160{,}850 \\ \underline{\qquad ? \qquad} & \text{if} \quad 160{,}850 < x \le 349{,}700 \\ \underline{\qquad ? \qquad} & \text{if} \quad x > 349{,}700. \end{cases}$$

Use this information to solve Exercises 91–94.

91. Find and interpret $T(20{,}000)$.

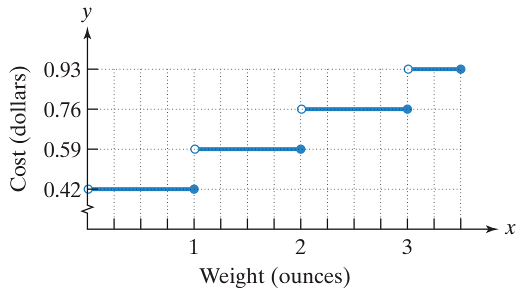
92. Find and interpret $T(50{,}000)$.

In Exercises 93–94, refer to the tax table on the previous page.

93. Find the algebraic expression for the missing piece of $T(x)$ that models tax owed for the domain (160,850, 349,700].

94. Find the algebraic expression for the missing piece of $T(x)$ that models tax owed for the domain $(349{,}700, \infty)$.

The figure shows the cost of mailing a first-class letter, $f(x)$, as a function of its weight, x, in ounces, for weights not exceeding 3.5 ounces. Use the graph to solve Exercises 95–98.



Source: Lynn E. Baring, Postmaster, Inverness, CA

95. Find $f(3)$. What does this mean in terms of the variables in this situation?

96. Find $f(3.5)$. What does this mean in terms of the variables in this situation?

97. What is the cost of mailing a letter that weighs 1.5 ounces?

98. What is the cost of mailing a letter that weighs 1.8 ounces?

99. If $3.5 < x \le 4$, the cost of mailing a first-class letter jumps to \$1.34. The cost then increases by \$0.17 per ounce for weights not exceeding 13 ounces:

Weight Not Exceeding	Cost
5 ounces	\$1.51
6 ounces	\$1.68

etc.

Use this information to extend the graph shown above so that the function's domain is (0, 13].

Writing in Mathematics

100. What does it mean if a function f is increasing on an interval?

101. Suppose that a function f whose graph contains no breaks or gaps on (a, c) is increasing on (a, b), decreasing on (b, c), and defined at b. Describe what occurs at $x = b$. What does the function value $f(b)$ represent?

102. If you are given a function's equation, how do you determine if the function is even, odd, or neither?

103. If you are given a function's graph, how do you determine if the function is even, odd, or neither?

104. What is a piecewise function?

105. Explain how to find the difference quotient of a function f, $\frac{f(x+h) - f(x)}{h}$, if an equation for f is given.

Technology Exercises

106. The function

$$f(x) = -0.00002x^3 + 0.008x^2 - 0.3x + 6.95$$

models the number of annual physician visits, $f(x)$, by a person of age x. Graph the function in a [0, 100, 5] by [0, 40, 2] viewing rectangle. What does the shape of the graph indicate about the relationship between one's age and the number of annual physician visits? Use the TABLE or minimum function capability to find the coordinates of the minimum point on the graph of the function. What does this mean?

In Exercises 107–112, use a graphing utility to graph each function. Use a $[-5, 5, 1]$ by $[-5, 5, 1]$ viewing rectangle. Then find the intervals on which the function is increasing, decreasing, or constant.

108. $f(x) = x^3 - 6x^2 + 9x + 1$

108. $g(x) = |4 - x^2|$

109. $h(x) = |x - 2| + |x + 2|$

110. $f(x) = x^{\frac{1}{3}}(x - 4)$

111. $g(x) = x^{\frac{2}{3}}$

112. $h(x) = 2 - x^{\frac{2}{5}}$

113. **a.** Graph the functions $f(x) = x^n$ for $n = 2, 4$, and 6 in a $[-2, 2, 1]$ by $[-1, 3, 1]$ viewing rectangle.

b. Graph the functions $f(x) = x^n$ for $n = 1, 3$, and 5 in a $[-2, 2, 1]$ by $[-2, 2, 1]$ viewing rectangle.

c. If n is positive and even, where is the graph of $f(x) = x^n$ increasing and where is it decreasing?

d. If n is positive and odd, what can you conclude about the graph of $f(x) = x^n$ in terms of increasing or decreasing behavior?

e. Graph all six functions in a $[-1, 3, 1]$ by $[-1, 3, 1]$ viewing rectangle. What do you observe about the graphs in terms of how flat or how steep they are?

Critical Thinking Exercises

Make Sense? *In Exercises 114–117, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

114. My graph is decreasing on $(-\infty, a)$ and increasing on (a, ∞), so $f(a)$ must be a relative maximum.

115. This work by artist Scott Kim (1955–) has the same kind of symmetry as an even function.

"DYSLEXIA," 1981

116. I graphed

$$f(x) = \begin{cases} 2 & \text{if } x \neq 4 \\ 3 & \text{if } x = 4 \end{cases}$$

and one piece of my graph is a single point.

117. I noticed that the difference quotient is always zero if $f(x) = c$, where c is any constant.

118. Sketch the graph of f using the following properties. (More than one correct graph is possible.) f is a piecewise function that is decreasing on $(-\infty, 2)$, $f(2) = 0$, f is increasing on $(2, \infty)$, and the range of f is $[0, \infty)$.

119. Define a piecewise function on the intervals $(-\infty, 2]$, $(2, 5)$, and $[5, \infty)$ that does not "jump" at 2 or 5 such that one piece is a constant function, another piece is an increasing function, and the third piece is a decreasing function.

120. Suppose that $h(x) = \frac{f(x)}{g(x)}$. The function f can be even, odd, or neither. The same is true for the function g.

a. Under what conditions is h definitely an even function?

b. Under what conditions is h definitely an odd function?

Group Exercise

121. (For assistance with this exercise, refer to the discussion of piecewise functions beginning on page 169, as well as to Exercises 79–80.) Group members who have cellular phone plans should describe the total monthly cost of the plan as follows:

> $_____ per month buys _____ minutes. Additional time costs $_____ per minute.

(For simplicity, ignore other charges.) The group should select any three plans, from "basic" to "premier." For each plan selected, write a piecewise function that describes the plan and graph the function. Graph the three functions in the same rectangular coordinate system. Now examine the graphs. For any given number of calling minutes, the best plan is the one whose graph is lowest at that point. Compare the three calling plans. Is one plan always a better deal than the other two? If not, determine the interval of calling minutes for which each plan is the best deal. (You can check out cellular phone plans by visiting www.point.com.)

Preview Exercises

Exercises 122–124 will help you prepare for the material covered in the next section.

122. If $(x_1, y_1) = (-3, 1)$ and $(x_2, y_2) = (-2, 4)$, find $\frac{y_2 - y_1}{x_2 - x_1}$.

123. Find the ordered pairs (_____, 0) and (0, _____) satisfying $4x - 3y - 6 = 0$.

124. Solve for y: $3x + 2y - 4 = 0$.

Section 1.4 Linear Functions and Slope

Objectives

1. Calculate a line's slope.
2. Write the point-slope form of the equation of a line.
3. Write and graph the slope-intercept form of the equation of a line.
4. Graph horizontal or vertical lines.
5. Recognize and use the general form of a line's equation.
6. Use intercepts to graph the general form of a line's equation.
7. Model data with linear functions and make predictions.

Is there a relationship between literacy and child mortality? As the percentage of adult females who are literate increases, does the mortality of children under five decrease? **Figure 1.35** on the next page indicates that this is, indeed, the case. Each point in the figure represents one country.