Chapter 3 Mid-Chapter Check Point

What You Know: We evaluated and graphed exponential functions $[f(x) = b^x, b > 0 \text{ and } b \neq 1]$, including the natural exponential function $[f(x) = e^x, e \approx 2.718]$. A function has an inverse that is a function if there is no horizontal line that intersects the function's graph more than once. The exponential function passes this horizontal line test and we called the inverse of the exponential function with base b the logarithmic function with base b. We learned that $y = \log_b x$ is equivalent to $b^y = x$. We evaluated and graphed logarithmic functions, including the common logarithmic function $[f(x) = \log_{10} x$ or $f(x) = \log x]$ and the natural logarithmic function $[f(x) = \log_e x$ or $f(x) = \ln x]$. We learned to use transformations to graph exponential and logarithmic functions. Finally, we used properties of logarithms to expand and condense logarithmic expressions.

In Exercises 1–5, graph f and g in the same rectangular coordinate system. Graph and give equations of all asymptotes. Give each function's domain and range.

1. $f(x) = 2^x$ and $g(x) = 2^x - 3$
2. $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \left(\frac{1}{2}\right)^{x-1}$
3. $f(x) = e^x$ and $g(x) = \ln x$
4. $f(x) = \log_2 x$ and $g(x) = \log_2(x - 1) + 1$
5. $g(x) = \log_{\frac{1}{2}} x$ and $g(x) = -2 \log_{\frac{1}{2}} x$

In Exercises 6–9, find the domain of each function.

6. $f(x) = \log_3(x + 6)$
7. $g(x) = \log_3 x + 6$
8. $h(x) = \log_3(x + 6)^2$
9. $f(x) = 3^{x+6}$

In Exercises 10–20, evaluate each expression without using a calculator. If evaluation is not possible, state the reason.

10. $\log_2 8 + \log_5 25$
11. $\log_3 \frac{1}{9}$
12. $\log_{100} 10$
13. $\log \sqrt[3]{10}$
14. $\log_2(\log_3 81)$
15. $\log_3\left(\log_2 \frac{1}{8}\right)$
16. $6^{\log_6 5}$
17. $\ln e^{\sqrt{7}}$
18. $10^{\log 13}$
19. $\log_{100} 0.1$
20. $\log_\pi \pi^{\sqrt{\pi}}$

In Exercises 21–22, expand and evaluate numerical terms.

21. $\log\left(\frac{\sqrt{xy}}{1000}\right)$
22. $\ln(e^{19}x^{20})$

In Exercises 23–25, write each expression as a single logarithm.

23. $8 \log_7 x - \frac{1}{3}\log_7 y$
24. $7 \log_5 x + 2 \log_5 x$
25. $\frac{1}{2}\ln x - 3 \ln y - \ln(z - 2)$
26. Use the formulas

$$A = P\left(1 + \frac{r}{n}\right)^{nt} \quad \text{and} \quad A = Pe^{rt}$$

to solve this exercise. You decide to invest \$8000 for 3 years at an annual rate of 8%. How much more is the return if the interest is compounded continuously than if it is compounded monthly? Round to the nearest dollar.

Section 3.4 Exponential and Logarithmic Equations

Objectives

1. Use like bases to solve exponential equations.
2. Use logarithms to solve exponential equations.
3. Use the definition of a logarithm to solve logarithmic equations.
4. Use the one-to-one property of logarithms to solve logarithmic equations.
5. Solve applied problems involving exponential and logarithmic equations.

At age 20, you inherit \$30,000. You'd like to put aside \$25,000 and eventually have over half a million dollars for early retirement. Is this possible? In this section, you will see how techniques for solving equations with variable exponents provide an answer to this question.

Exponential Equations

An **exponential equation** is an equation containing a variable in an exponent. Examples of exponential equations include

$$2^{3x-8} = 16, \qquad 4^x = 15, \quad \text{and} \quad 40e^{0.6x} = 240.$$

1 Use like bases to solve exponential equations.

Some exponential equations can be solved by expressing each side of the equation as a power of the same base. All exponential functions are one-to-one—that is, no two different ordered pairs have the same second component. Thus, if b is a positive number other than 1 and $b^M = b^N$, then $M = N$.