Chapter 4 Summary, Review, and Test

Summary

DEFINITIONS AND CONCEPTS	EXAMPLES
4.1 Angles and Radian Measure	
a. An angle consists of two rays with a common endpoint, the vertex.	
b. An angle is in standard position if its vertex is at the origin and its initial side lies along the positive x-axis. Figure 4.3 on page 461 shows positive and negative angles in standard position.	
c. A quadrantal angle is an angle with its terminal side on the x-axis or the y-axis.	
d. Angles can be measured in degrees. $1°$ is $\frac{1}{360}$ of a complete rotation.	
e. Acute angles measure more than $0°$ but less than $90°$, right angles $90°$, obtuse angles more than $90°$ but less than $180°$, and straight angles $180°$.	Figure 4.5, p. 461
f. Angles can be measured in radians. One radian is the measure of the central angle when the intercepted arc and radius have the same length. In general, the radian measure of a central angle is the length of the intercepted arc divided by the circle's radius: $\theta = \frac{s}{r}$.	Ex. 1, p. 463
g. To convert from degrees to radians, multiply degrees by $\frac{\pi \text{ radians}}{180°}$. To convert from radians to degrees, multiply radians by $\frac{180°}{\pi \text{ radians}}$.	Ex. 2, p. 464; Ex. 3, p. 464
h. To draw angles measured in radians in standard position, it is helpful to "think in radians" without having to convert to degrees. See Figure 4.15 on page 467.	Ex. 4, p. 465
i. Two angles with the same initial and terminal sides are called coterminal angles. Increasing or decreasing an angle's measure by integer multiples of $360°$ or 2π produces coterminal angles.	Ex. 5, p. 468; Ex. 6, p. 469; Ex. 7, p. 469
j. The arc length formula, $s = r\theta$, is described in the box on page 470.	Ex. 8, p. 470
k. The definitions of linear speed, $v = \frac{s}{t}$, and angular speed, $\omega = \frac{\theta}{t}$, are given in the box on page 471.	
l. Linear speed is expressed in terms of angular speed by $v = r\omega$, where v is the linear speed of a point a distance r from the center of rotation and ω is the angular speed in radians per unit of time.	Ex. 9, p. 472
4.2 Trigonometric Functions: The Unit Circle	
a. Definitions of the trigonometric functions in terms of a unit circle are given in the box on page 477.	Ex. 1, p. 477; Ex. 2, p. 478
b. The cosine and secant functions are even: $\cos(-t) = \cos t, \quad \sec(-t) = \sec t.$ The other trigonometric functions are odd: $\sin(-t) = -\sin t, \quad \csc(-t) = -\csc t,$ $\tan(-t) = -\tan t, \quad \cot(-t) = -\cot t.$	Ex. 4, p. 481
c. Fundamental Identities **1.** Reciprocal Identities $\sin t = \frac{1}{\csc t}$ and $\csc t = \frac{1}{\sin t}$; $\cos t = \frac{1}{\sec t}$ and $\sec t = \frac{1}{\cos t}$; $\tan t = \frac{1}{\cot t}$ and $\cot t = \frac{1}{\tan t}$ **2.** Quotient Identities $\tan t = \frac{\sin t}{\cos t}$; $\cot t = \frac{\cos t}{\sin t}$	Ex. 5, p. 482; Ex. 6, p. 483

DEFINITIONS AND CONCEPTS | **EXAMPLES**

3. Pythagorean Identities

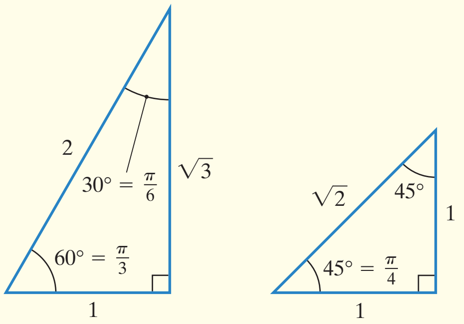
$$\sin^2 t + \cos^2 t = 1; 1 + \tan^2 t = \sec^2 t; 1 + \cot^2 t = \csc^2 t$$

d. If $f(t + p) = f(t)$, the function f is periodic. The smallest positive value of p for which $f(t + p) = f(t)$ is the period of f. The tangent and cotangent functions have period π. The other four trigonometric functions have period 2π. — Ex. 7, p. 484

4.3 Right Triangle Trigonometry

a. The right triangle definitions of the six trigonometric functions are given in the box on page 490. — Ex. 1, p. 491; Ex. 2, p. 492

b. Function values for 30°, 45°, and 60° can be obtained using these special triangles. — Ex. 3, p. 493; Ex. 4, p. 493



c. Two angles are complements if their sum is 90° or $\frac{\pi}{2}$. The value of a trigonometric function of θ is equal to the cofunction of the complement of θ. Cofunction identities are listed in the box on page 495. — Ex. 5, p. 495

4.4 Trigonometric Functions of Any Angle

a. Definitions of the trigonometric functions of any angle are given in the box on page 502. — Ex. 1, p. 503; Ex. 2, p. 503

b. Signs of the trigonometric functions: All functions are positive in quadrant I. If θ lies in quadrant II, $\sin\theta$ and $\csc\theta$ are positive. If θ lies in quadrant III, $\tan\theta$ and $\cot\theta$ are positive. If θ lies in quadrant IV, $\cos\theta$ and $\sec\theta$ are positive. — Ex. 3, p. 505; Ex. 4, p. 505

c. If θ is a nonacute angle in standard position that lies in a quadrant, its reference angle is the positive acute angle θ' formed by the terminal side of θ and the x-axis. The reference angle for a given angle can be found by making a sketch that shows the angle in standard position. Figure 4.49 on page 506 shows reference angles for θ in quadrants II, III, and IV. — Ex. 5, p. 506; Ex. 6, p. 507

d. The values of the trigonometric functions of a given angle are the same as the values of the functions of the reference angle, except possibly for the sign. A procedure for using reference angles to evaluate trigonometric functions is given in the box on page 508. — Ex. 7, p. 509; Ex. 8, p. 510

4.5 and 4.6 Graphs of the Trigonometric Functions

a. Graphs of the six trigonometric functions, with a description of the domain, range, and period of each function, are given in Table 4.6 on page 545.

b. The graph of $y = A\sin(Bx - C)$ can be obtained using amplitude $= |A|$, period $= \frac{2\pi}{B}$, and phase shift $= \frac{C}{B}$. See the illustration in the box on page 523. — Ex. 1, p. 518; Ex. 2, p. 519; Ex. 3, p. 521; Ex. 4, p. 523

DEFINITIONS AND CONCEPTS	EXAMPLES
c. The graph of $y = A\cos(Bx - C)$ can be obtained using amplitude $= \lvert A \rvert$, period $= \frac{2\pi}{B}$, and phase shift $= \frac{C}{B}$. See the illustration in the box on page 528.	Ex. 5, p. 526; Ex. 6, p. 528
d. The constant D in $y = A\sin(Bx - C) + D$ and $y = A\cos(Bx - C) + D$ causes vertical shifts in the graphs in the preceding items (b) and (c). If $D > 0$, the shift is D units upward and if $D < 0$, the shift is $\lvert D \rvert$ units downward. Oscillation is about the horizontal line $y = D$.	Ex. 7, p. 529
e. The graph of $y = A\tan(Bx - C)$ is obtained using the procedure in the box on page 539. Consecutive asymptotes $\left(\text{solve } -\frac{\pi}{2} < Bx - C < \frac{\pi}{2}\text{; consecutive asymptotes occur at } Bx - C = -\frac{\pi}{2} \text{ and } Bx - C = \frac{\pi}{2}\right)$ and an x-intercept midway between them play a key role in the graphing process.	Ex. 1, p. 539; Ex. 2, p. 540
f. The graph of $y = A\cot(Bx - C)$ is obtained using the procedure in the lower box on page 541. Consecutive asymptotes (solve $0 < Bx - C < \pi$; consecutive asymptotes occur at $Bx - C = 0$ and $Bx - C = \pi$) and an x-intercept midway between them play a key role in the graphing process.	Ex. 3, p. 541
g. To graph a cosecant curve, begin by graphing the corresponding sine curve. Draw vertical asymptotes through x-intercepts, using asymptotes as guides to sketch the graph. To graph a secant curve, first graph the corresponding cosine curve and use the same procedure.	Ex. 4, p. 543; Ex. 5, p. 544
4.7 Inverse Trigonometric Functions	
a. On the restricted domain $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, $y = \sin x$ has an inverse function, defined in the box on page 551. Think of $\sin^{-1} x$ as the angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is x. A procedure for finding exact values of $\sin^{-1} x$ is given in the box on page 552.	Ex. 1, p. 552; Ex. 2, p. 553
b. On the restricted domain $0 \le x \le \pi$, $y = \cos x$ has an inverse function, defined in the upper box on page 554. Think of $\cos^{-1} x$ as the angle in $[0, \pi]$ whose cosine is x. A procedure for finding exact values of $\cos^{-1} x$ is given in the lower box on page 554.	Ex. 3, p. 554
c. On the restricted domain $-\frac{\pi}{2} < x < \frac{\pi}{2}$, $y = \tan x$ has an inverse function, defined in the box on page 555. Think of $\tan^{-1} x$ as the angle in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ whose tangent is x. A procedure for finding exact values of $\tan^{-1} x$ is given in the box on page 556.	Ex. 4, p. 556
d. Graphs of the three basic inverse trigonometric functions, with a description of the domain and range of each function, are given in Table 4.10 on page 557.	
e. Inverse properties are given in the box on page 559. Points on terminal sides of angles in standard position and right triangles are used to find exact values of the composition of a function and a different inverse function.	Ex. 6, p. 560; Ex. 7, p. 560; Ex. 8, p. 561; Ex. 9, p. 562
4.8 Applications of Trigonometric Functions	
a. Solving a right triangle means finding the missing lengths of its sides and the measurements of its angles. The Pythagorean Theorem, two acute angles whose sum is 90°, and appropriate trigonometric functions are used in this process.	Ex. 1, p. 566; Ex. 2, p. 567; Ex. 3, p. 567; Ex. 4, p. 568
b. The bearing from point O to point P is the acute angle between ray OP and a north-south line, shown in Figure 4.105 on page 569.	Ex. 5, p. 569; Ex. 6, p. 570
c. Simple harmonic motion, described in the box on page 571, is modeled by $d = a\cos\omega t$ or $d = a\sin\omega t$, with amplitude $= \lvert a \rvert$, period $= \frac{2\pi}{\omega}$, and frequency $= \frac{\omega}{2\pi} = \frac{1}{\text{period}}$.	Ex. 7, p. 571; Ex. 8, p. 572

Study Tip

Much of the essential information in this chapter can be found in three places:

- Study Tip on page 512, showing special angles and how to obtain exact values of trigonometric functions at these angles
- **Table 4.6** on page 545, showing the graphs of the six trigonometric functions, with their domains, ranges, and periods
- **Table 4.10** on page 557, showing graphs of the three basic inverse trigonometric functions, with their domains and ranges.

Make copies of these pages and mount them on cardstock. Use this reference sheet as you work the review exercises until you have all the information on the reference sheet memorized for the chapter test.

Review Exercises

4.1

1. Find the radian measure of the central angle of a circle of radius 6 centimeters that intercepts an arc of length 27 centimeters.

In Exercises 2–4, convert each angle in degrees to radians. Express your answer as a multiple of π.

2. $15°$ **3.** $120°$ **4.** $315°$

In Exercises 5–7, convert each angle in radians to degrees.

5. $\frac{5\pi}{3}$ **6.** $\frac{7\pi}{5}$ **7.** $-\frac{5\pi}{6}$

In Exercises 8–12, draw each angle in standard position.

8. $\frac{5\pi}{6}$ **9.** $-\frac{2\pi}{3}$ **10.** $\frac{8\pi}{3}$

11. $190°$ **12.** $-135°$

In Exercises 13–17, find a positive angle less than 360° or 2π that is coterminal with the given angle.

13. $400°$ **14.** $-445°$ **15.** $\frac{13\pi}{4}$

16. $\frac{31\pi}{6}$ **17.** $-\frac{8\pi}{3}$

18. Find the length of the arc on a circle of radius 10 feet intercepted by a 135° central angle. Express arc length in terms of π. Then round your answer to two decimal places.

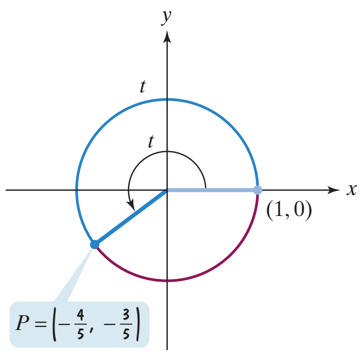
19. The angular speed of a propeller on a wind generator is 10.3 revolutions per minute. Express this angular speed in radians per minute.

20. The propeller of an airplane has a radius of 3 feet. The propeller is rotating at 2250 revolutions per minute. Find the linear speed, in feet per minute, of the tip of the propeller.

4.2

In Exercises 21–22, a point $P(x, y)$ is shown on the unit circle corresponding to a real number t. Find the values of the trigonometric functions at t.

21.



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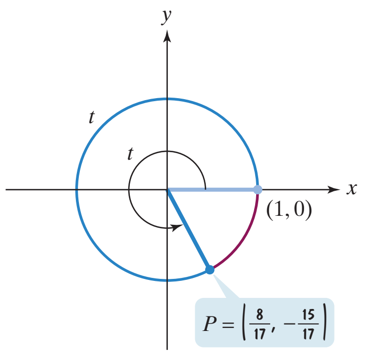
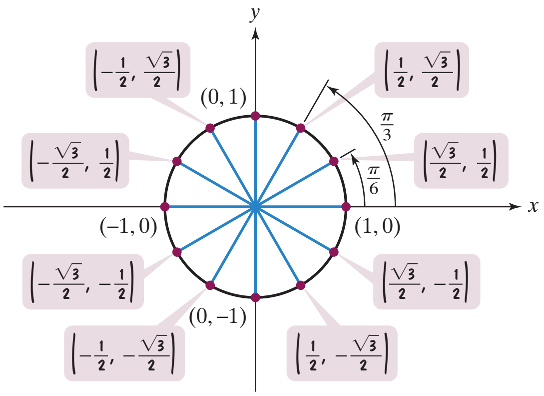


In Exercises 23–26, use the figure shown to find the value of each trigonometric function at the indicated real number or state that the expression is undefined.



23. $\sec \frac{5\pi}{6}$ **24.** $\tan \frac{4\pi}{3}$

25. $\sec \frac{\pi}{2}$ **26.** $\cot \pi$

27. If $\sin t = \frac{2\sqrt{7}}{7}, 0 \le t < \frac{\pi}{2}$, use identities to find the remaining trigonometric functions.

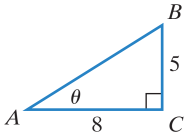
In Exercises 28–30 evaluate each expression without using a calculator.

28. $\tan 4.7 \cot 4.7$ **29.** $\sin^2 \frac{\pi}{17} + \cos^2 \frac{\pi}{17}$

30. $\tan^2 1.4 - \sec^2 1.4$

4.3

31. Use the triangle to find each of the six trigonometric functions of θ.



In Exercises 32–35, find the exact value of each expression. Do not use a calculator.

32. $\sin\frac{\pi}{6} + \tan^2\frac{\pi}{3}$

33. $\cos^2\frac{\pi}{4} - \tan^2\frac{\pi}{4}$

34. $\sec^2\frac{\pi}{5} - \tan^2\frac{\pi}{5}$

35. $\cos\frac{2\pi}{9}\sec\frac{2\pi}{9}$

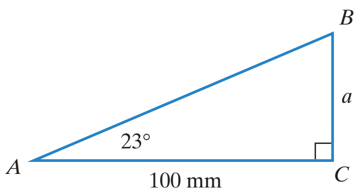
In Exercises 36–37, find a cofunction with the same value as the given expression.

36. $\sin 70°$

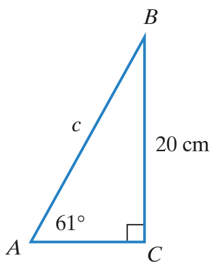
37. $\cos\frac{\pi}{2}$

In Exercises 38–40, find the measure of the side of the right triangle whose length is designated by a lowercase letter. Round answers to the nearest whole number.

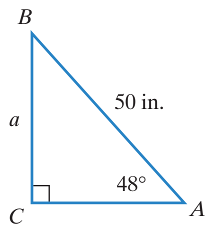
38.



39.



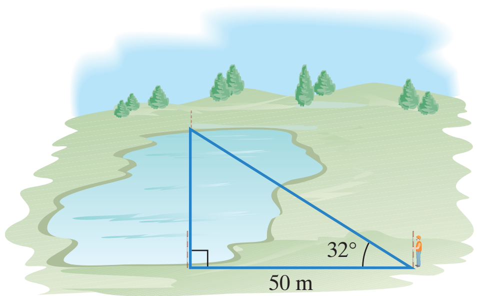
40.



41. If $\sin\theta = \frac{1}{4}$ and θ is acute, find $\tan\left(\frac{\pi}{2} - \theta\right)$.

42. A hiker climbs for a half mile up a slope whose inclination is 17°. How many feet of altitude, to the nearest foot, does the hiker gain?

43. To find the distance across a lake, a surveyor took the measurements in the figure shown. What is the distance across the lake? Round to the nearest meter.



44. When a six-foot pole casts a four-foot shadow, what is the angle of elevation of the sun? Round to the nearest whole degree.

4.4

In Exercises 45–46, a point on the terminal side of angle θ is given. Find the exact value of each of the six trigonometric functions of θ, or state that the function is undefined.

45. $(-1, -5)$

46. $(0, -1)$

In Exercises 47–48, let θ be an angle in standard position. Name the quadrant in which θ lies.

47. $\tan\theta > 0$ and $\sec\theta > 0$

48. $\tan\theta > 0$ and $\cos\theta < 0$

In Exercises 49–51, find the exact value of each of the remaining trigonometric functions of θ.

49. $\cos\theta = \frac{2}{5}$, $\sin\theta < 0$

50. $\tan\theta = -\frac{1}{3}$, $\sin\theta > 0$

51. $\cot\theta = 3$, $\cos\theta < 0$

In Exercises 52–56, find the reference angle for each angle.

52. $265°$

53. $\frac{5\pi}{8}$

54. $-410°$

55. $\frac{17\pi}{6}$

56. $-\frac{11\pi}{3}$

In Exercises 57–67, find the exact value of each expression. Do not use a calculator.

57. $\sin 240°$

58. $\tan 120°$

59. $\sec\frac{7\pi}{4}$

60. $\cos\frac{11\pi}{6}$

61. $\cot(-210°)$

62. $\csc\left(-\frac{2\pi}{3}\right)$

63. $\sin\left(-\frac{\pi}{3}\right)$

64. $\sin 495°$

65. $\tan\frac{13\pi}{4}$

66. $\sin\frac{22\pi}{3}$

67. $\cos\left(-\frac{35\pi}{6}\right)$

4.5

In Exercises 68–73, determine the amplitude and period of each function. Then graph one period of the function.

68. $y = 3\sin 4x$

69. $y = -2\cos 2x$

70. $y = 2\cos\frac{1}{2}x$

71. $y = \frac{1}{2}\sin\frac{\pi}{3}x$

72. $y = -\sin\pi x$

73. $y = 3\cos\frac{x}{3}$

In Exercises 74–78, determine the amplitude, period, and phase shift of each function. Then graph one period of the function.

74. $y = 2\sin(x - \pi)$

75. $y = -3\cos(x + \pi)$

76. $y = \frac{3}{2}\cos\left(2x + \frac{\pi}{4}\right)$

77. $y = \frac{5}{2}\sin\left(2x + \frac{\pi}{2}\right)$

78. $y = -3\sin\left(\frac{\pi}{3}x - 3\pi\right)$

In Exercises 79–80, use a vertical shift to graph one period of the function.

79. $y = \sin 2x + 1$

80. $y = 2\cos\frac{1}{3}x - 2$

81. The function

$$y = 98.6 + 0.3 \sin\left(\frac{\pi}{12}x - \frac{11\pi}{12}\right)$$

models variation in body temperature, y, in °F, x hours after midnight.

a. What is body temperature at midnight?

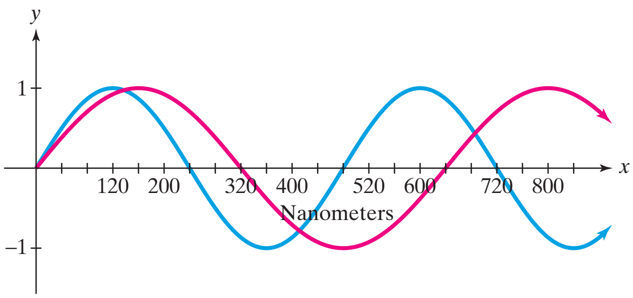
b. What is the period of the body temperature cycle?

c. When is body temperature highest? What is the body temperature at this time?

d. When is body temperature lowest? What is the body temperature at this time?

e. Graph one period of the body temperature function.

82. Light waves can be modeled by sine functions. The graphs show waves of red and blue light. Write an equation in the form $y = A \sin Bx$ that models each of these light waves.



4.6

In Exercises 83–89, graph two full periods of the given tangent or cotangent function.

83. $y = 4 \tan 2x$

84. $y = -2 \tan \frac{\pi}{4}x$

85. $y = \tan(x + \pi)$

86. $y = -\tan\left(x - \frac{\pi}{4}\right)$

87. $y = 2 \cot 3x$

88. $y = -\frac{1}{2} \cot \frac{\pi}{2}x$

90. $y = 2 \cot\left(x + \frac{\pi}{2}\right)$

In Exercises 90–93, graph two full periods of the given cosecant or secant function.

90. $y = 3 \sec 2\pi x$

91. $y = -2 \csc \pi x$

92. $y = 3 \sec(x + \pi)$

93. $y = \frac{5}{2} \csc(x - \pi)$

4.7

In Exercises 94–112, find the exact value of each expression. Do not use a calculator.

94. $\sin^{-1} 1$

95. $\cos^{-1} 1$

96. $\tan^{-1} 1$

97. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

98. $\cos^{-1}\left(-\frac{1}{2}\right)$

99. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

100. $\cos\left(\sin^{-1} \frac{\sqrt{2}}{2}\right)$

101. $\sin(\cos^{-1} 0)$

102. $\tan\left[\sin^{-1}\left(-\frac{1}{2}\right)\right]$

103. $\tan\left[\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

104. $\csc\left(\tan^{-1} \frac{\sqrt{3}}{3}\right)$

105. $\cos\left(\tan^{-1} \frac{3}{4}\right)$

106. $\sin\left(\cos^{-1} \frac{3}{5}\right)$

107. $\tan\left[\sin^{-1}\left(-\frac{3}{5}\right)\right]$

108. $\tan\left[\cos^{-1}\left(-\frac{4}{5}\right)\right]$

109. $\sin\left[\tan^{-1}\left(-\frac{1}{3}\right)\right]$

110. $\sin^{-1}\left(\sin \frac{\pi}{3}\right)$

111. $\sin^{-1}\left(\sin \frac{2\pi}{3}\right)$

112. $\sin^{-1}\left(\cos \frac{2\pi}{3}\right)$

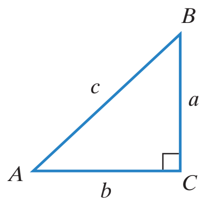
In Exercises 113–114, use a right triangle to write each expression as an algebraic expression. Assume that x is positive and that the given inverse trigonometric function is defined for the expression in x.

113. $\cos\left(\tan^{-1} \frac{x}{2}\right)$

114. $\sec\left(\sin^{-1} \frac{1}{x}\right)$

4.8

In Exercises 115–118, solve the right triangle shown in the figure. Round lengths to two decimal places and express angles to the nearest tenth of a degree.



115. $A = 22.3°, c = 10$

116. $B = 37.4°, b = 6$

117. $a = 2, c = 7$

118. $a = 1.4, b = 3.6$

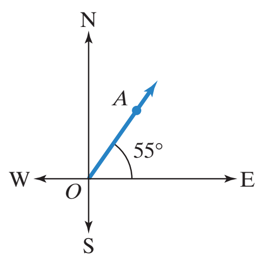
119. From a point on level ground 80 feet from the base of a building, the angle of elevation is 25.6°. Approximate the height of the building to the nearest foot.

120. Two buildings with flat roofs are 60 yards apart. The height of the shorter building is 40 yards. From its roof, the angle of elevation to the edge of the roof of the taller building is 40°. Find the height of the taller building to the nearest yard.

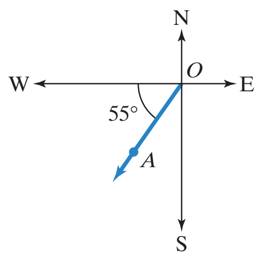
121. You want to measure the height of an antenna on the top of a 125-foot building. From a point in front of the building, you measure the angle of elevation to the top of the building to be 68° and the angle of elevation to the top of the antenna to be 71°. How tall is the antenna, to the nearest tenth of a foot?

In Exercises 122–123, use the figures shown to find the bearing from O to A.

122.



123.



124. A ship is due west of a lighthouse. A second ship is 12 miles south of the first ship. The bearing from the second ship to the lighthouse is N 64° E. How far, to the nearest tenth of a mile, is the first ship from the lighthouse?

125. From city A to city B, a plane flies 850 miles at a bearing of N 58° E. From city B to city C, the plane flies 960 miles at a bearing of S 32° E.

a. Find, to the nearest tenth of a mile, the distance from city A to city C.

b. What is the bearing from city A to city C?

In Exercises 126–127, an object moves in simple harmonic motion described by the given equation, where t is measured in seconds and d in centimeters. In each exercise, find:

a. the maximum displacement

b. the frequency

c. the time required for one cycle.

126. $d = 20 \cos \frac{\pi}{4} t$ **127.** $d = \frac{1}{2} \sin 4t$

In Exercises 128–129, an object is attached to a coiled spring. In Exercise 128, the object is pulled down (negative direction from the rest position) and then released. In Exercise 129, the object is propelled downward from its rest position at time $t = 0$. Write an equation for the distance of the object from its rest position after t seconds.

Distance from Rest Position at $t = 0$	Amplitude	Period
128. 30 inches	30 inches	2 seconds
129. 0 inches	$\frac{1}{4}$ inch	5 seconds

CHAPTER Test Prep VIDEOS

Chapter 4 Test

1. Convert 135° to an exact radian measure.

2. Find the length of the arc on a circle of radius 20 feet intercepted by a 75° central angle. Express arc length in terms of π. Then round your answer to two decimal places.

3. a. Find a positive angle less than 2π that is coterminal with $\frac{16\pi}{3}$.

b. Find the reference angle for $\frac{16\pi}{3}$.

4. If $(-2, 5)$ is a point on the terminal side of angle θ, find the exact value of each of the six trigonometric functions of θ.

5. Determine the quadrant in which θ lies if $\cos \theta < 0$ and $\cot \theta > 0$.

6. If $\cos \theta = \frac{1}{3}$ and $\tan \theta < 0$, find the exact value of each of the remaining trigonometric functions of θ.

In Exercises 7–12, find the exact value of each expression. Do not use a calculator.

7. $\tan \frac{\pi}{6} \cos \frac{\pi}{3} - \cos \frac{\pi}{2}$ **8.** $\tan 300°$

9. $\sin \frac{7\pi}{4}$ **10.** $\sec \frac{22\pi}{3}$

11. $\cot\left(-\frac{8\pi}{3}\right)$

12. $\tan\left(\frac{7\pi}{3} + n\pi\right)$, n is an integer.

13. If $\sin \theta = a$ and $\cos \theta = b$, represent each of the following in terms of a and b.

a. $\sin(-\theta) + \cos(-\theta)$ **b.** $\tan \theta - \sec \theta$

In Exercises 14–17, graph one period of each function.

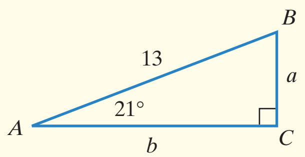
14. $y = 3 \sin 2x$ **15.** $y = -2 \cos\left(x - \frac{\pi}{2}\right)$

16. $y = 2 \tan \frac{x}{2}$ **17.** $y = -\frac{1}{2} \csc \pi x$

18. Find the exact value of $\tan\left[\cos^{-1}\left(-\frac{1}{2}\right)\right]$.

19. Write $\sin\left(\cos^{-1} \frac{x}{3}\right)$ as an algebraic expression. Assume that $x > 0$ and $\frac{x}{3}$ is in the domain of the inverse cosine function.

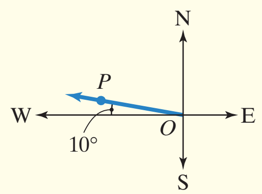
20. Solve the right triangle in the figure shown. Round lengths to one decimal place.



21. The angle of elevation to the top of a building from a point on the ground 30 yards from its base is 37°. Find the height of the building to the nearest yard.

22. A 73-foot rope from the top of a circus tent pole is anchored to the flat ground 43 feet from the bottom of the pole. Find the angle, to the nearest tenth of a degree, that the rope makes with the pole.

23. Use the figure to find the bearing from O to P.



24. An object moves in simple harmonic motion described by $d = -6 \cos \pi t$, where t is measured in seconds and d in inches. Find **a.** the maximum displacement, **b.** the frequency, and **c.** the time required for one oscillation.

25. Why are trigonometric functions ideally suited to model phenomena that repeat in cycles?

Cumulative Review Exercises (Chapters P–4)

Solve each equation or inequality in Exercises 1–6.

1. $x^2 = 18 + 3x$
2. $x^3 + 5x^2 - 4x - 20 = 0$
3. $\log_2 x + \log_2(x - 2) = 3$
4. $\sqrt{x - 3} + 5 = x$
5. $x^3 - 4x^2 + x + 6 = 0$
6. $|2x - 5| \le 11$
7. If $f(x) = \sqrt{x - 6}$, find $f^{-1}(x)$.
8. Divide $20x^3 - 6x^2 - 9x + 10$ by $5x + 2$.
9. Write as a single logarithm and evaluate: $\log 25 + \log 40$.
10. Convert $\frac{14\pi}{9}$ radians to degrees.
11. Find the maximum number of positive and negative real roots of the equation $3x^4 - 2x^3 + 5x^2 + x - 9 = 0$.

In Exercises 12–16, graph each equation.

12. $f(x) = \frac{x}{x^2 - 1}$
13. $(x - 2)^2 + y^2 = 1$
14. $y = (x - 1)(x + 2)^2$
15. $y = \sin\left(2x + \frac{\pi}{2}\right)$, from 0 to 2π
16. $y = 2 \tan 3x$; Graph two complete cycles.
17. You invest in a new play. The cost includes an overhead of \$30,000, plus production costs of \$2500 per performance. A sold-out performance brings you \$3125. How many sold-out performances must be played in order for you to break even?
18. Use the exponential growth model $A = A_0 e^{kt}$ to solve this exercise. In 2000, there were 110 million cell phone subscribers in the United States. By 2006, there were 233 million subscribers. (*Source:* CTIA)
    a. Find the exponential function that models the data.
    b. According to the model, in which year were there 300 million cell phone subscribers in the United States?
19. The rate of heat lost through insulation varies inversely as the thickness of the insulation. The rate of heat lost through a 3.5-inch thickness of insulation is 2200 Btu per hour. What is the rate of heat lost through a 5-inch thickness of the same insulation?
20. A tower is 200 feet tall. To the nearest degree, find the angle of elevation from a point 50 feet from the base of the tower to the top of the tower.