Preview Exercises

Exercises 163–165 will help you prepare for the material covered in the first section of the next chapter. Solve each equation by using the cross-products principle to clear fractions from the proportion:

$$\text{If } \frac{a}{b} = \frac{c}{d}, \text{ then } ad = bc. \ (b \neq 0 \text{ and } d \neq 0)$$

Round to the nearest tenth.

163. Solve for a: $\frac{a}{\sin 46°} = \frac{56}{\sin 63°}$.

164. Solve for B, $0 < B < 180°$: $\frac{81}{\sin 43°} = \frac{62}{\sin B}$.

165. Solve for B: $\frac{51}{\sin 75°} = \frac{71}{\sin B}$.

Chapter 5 Summary, Review, and Test

Summary

DEFINITIONS AND CONCEPTS	EXAMPLES
5.1 Verifying Trigonometric Identities	
a. Identities are trigonometric equations that are true for all values of the variable for which the expressions are defined.	
b. Fundamental trigonometric identities are given in the box on page 586.	
c. Guidelines for verifying trigonometric identities are given in the box on page 593.	Ex. 1, p. 587; Ex. 2, p. 588; Ex. 3, p. 588; Ex. 4, p. 589; Ex. 5, p. 590; Ex. 6, p. 590; Ex. 7, p. 591; Ex. 8, p. 592
5.2 Sum and Difference Formulas	
a. Sum and difference formulas are given in the box on page 599 and the box on page 603.	
b. Sum and difference formulas can be used to find exact values of trigonometric functions.	Ex. 1, p. 598; Ex. 2, p. 598; Ex. 4, p. 600; Ex. 5, p. 601
c. Sum and difference formulas can be used to verify trigonometric identities.	Ex. 3, p. 599; Ex. 6, p. 602; Ex. 7, p. 603
5.3 Double-Angle, Power-Reducing, and Half-Angle Formulas	
a. Double-angle, power-reducing, and half-angle formulas are given in the box on page 614.	
b. Double-angle and half-angle formulas can be used to find exact values of trigonometric functions.	Ex. 1, p. 608; Ex. 2, p. 609; Ex. 5, p. 612
c. Double-angle and half-angle formulas can be used to verify trigonometric identities.	Ex. 3, p. 609; Ex. 6, p. 613; Ex. 7, p. 613
d. Power-reducing formulas can be used to reduce the powers of trigonometric functions.	Ex. 4, p. 610
5.4 Product-to-Sum and Sum-to-Product Formulas	
a. The product-to-sum formulas are given in the box on page 619.	Ex. 1, p. 620
b. The sum-to-product formulas are given in the box on page 620. These formulas are useful to verify identities with fractions that contain sums and differences of sines and/or cosines.	Ex. 2, p. 621; Ex. 3, p. 621

DEFINITIONS AND CONCEPTS	EXAMPLES
5.5 Trigonometric Equations	
a. The values that satisfy a trigonometric equation are its solutions.	
b. To solve an equation containing a single trigonometric function, isolate the function on one side and solve for the variable.	Ex. 1, p. 627
c. When solving equations involving multiple angles, the period plays an important role in ensuring that we do not leave out any solutions.	Ex. 2, p. 628; Ex. 3, p. 628
d. Trigonometric equations quadratic in form can be expressed as $au^2 + bu + c = 0$, where u is a trigonometric function and $a \neq 0$. Such equations can be solved by factoring, the square root property, or the quadratic formula.	Ex. 4, p. 629; Ex. 5, p. 630; Ex. 12, p. 635
e. Factoring can be used to separate two different trigonometric functions in an equation.	Ex. 6, p. 631
f. Identities are used to solve some trigonometric equations.	Ex. 7, p. 631; Ex. 8, p. 632; Ex. 9, p. 633; Ex. 10, p. 633
g. Some trigonometric equations have solutions that cannot be determined by knowing the exact values of trigonometric functions of special angles. Such equations are solved using a calculator's inverse trigonometric function feature.	Ex. 11, p. 634; Ex. 12, p. 635

Review Exercises

5.1

In Exercises 1–13, verify each identity.

1. $\sec x - \cos x = \tan x \sin x$

2. $\cos x + \sin x \tan x = \sec x$

3. $\sin^2 \theta(1 + \cot^2 \theta) = 1$

4. $(\sec \theta - 1)(\sec \theta + 1) = \tan^2 \theta$

5. $\dfrac{1 - \tan x}{\sin x} = \csc x - \sec x$

6. $\dfrac{1}{\sin t - 1} + \dfrac{1}{\sin t + 1} = -2 \tan t \sec t$

7. $\dfrac{1 + \sin t}{\cos^2 t} = \tan^2 t + 1 + \tan t \sec t$

8. $\dfrac{\cos x}{1 - \sin x} = \dfrac{1 + \sin x}{\cos x}$

9. $1 - \dfrac{\sin^2 x}{1 + \cos x} = \cos x$

10. $(\tan \theta + \cot \theta)^2 = \sec^2 \theta + \csc^2 \theta$

11. $\dfrac{1}{\sin \theta + \cos \theta} + \dfrac{1}{\sin \theta - \cos \theta} = \dfrac{2 \sin \theta}{\sin^4 \theta - \cos^4 \theta}$

12. $\dfrac{\cos t}{\cot t - 5 \cos t} = \dfrac{1}{\csc t - 5}$

13. $\dfrac{1 - \cos t}{1 + \cos t} = (\csc t - \cot t)^2$

5.2 and 5.3

In Exercises 14–19, use a sum or difference formula to find the exact value of each expression.

14. $\cos(45° + 30°)$

15. $\sin 195°$

16. $\tan\left(\dfrac{4\pi}{3} - \dfrac{\pi}{4}\right)$

17. $\tan \dfrac{5\pi}{12}$

18. $\cos 65° \cos 5° + \sin 65° \sin 5°$

19. $\sin 80° \cos 50° - \cos 80° \sin 50°$

In Exercises 20–31, verify each identity.

20. $\sin\left(x + \dfrac{\pi}{6}\right) - \cos\left(x + \dfrac{\pi}{3}\right) = \sqrt{3} \sin x$

21. $\tan\left(x + \dfrac{3\pi}{4}\right) = \dfrac{\tan x - 1}{1 + \tan x}$

22. $\sec(\alpha + \beta) = \dfrac{\sec \alpha \sec \beta}{1 - \tan \alpha \tan \beta}$

23. $\dfrac{\cos(\alpha - \beta)}{\cos \alpha \cos \beta} = 1 + \tan \alpha \tan \beta$

24. $\cos^4 t - \sin^4 t = \cos 2t$

25. $\sin t - \cos 2t = (2 \sin t - 1)(\sin t + 1)$

26. $\dfrac{\sin 2\theta - \sin \theta}{\cos 2\theta + \cos \theta} = \dfrac{1 - \cos \theta}{\sin \theta}$

27. $\dfrac{\sin 2\theta}{1 - \sin^2 \theta} = 2 \tan \theta$

28. $\tan 2t = 2 \sin t \cos t \sec 2t$

29. $\cos 4t = 1 - 8 \sin^2 t \cos^2 t$

30. $\tan \dfrac{x}{2}(1 + \cos x) = \sin x$

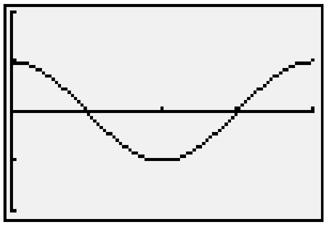
31. $\tan \dfrac{x}{2} = \dfrac{\sec x - 1}{\tan x}$

In Exercises 32–34, the graph with the given equation is shown in a $\left[0, 2\pi, \dfrac{\pi}{2}\right]$ by $[-2, 2, 1]$ viewing rectangle.

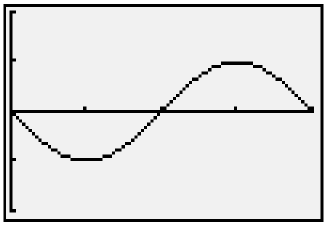
a. *Describe the graph using another equation.*

b. *Verify that the two equations are equivalent.*

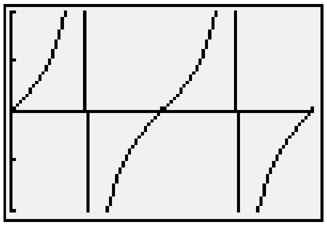
32. $y = \sin\left(x - \frac{3\pi}{2}\right)$



33. $y = \cos\left(x + \frac{\pi}{2}\right)$



34. $y = \frac{\tan x - 1}{1 - \cot x}$



In Exercises 35–38, find the exact value of the following under the given conditions:

a. $\sin(\alpha + \beta)$

b. $\cos(\alpha - \beta)$

c. $\tan(\alpha + \beta)$

d. $\sin 2\alpha$

e. $\cos\frac{\beta}{2}$

35. $\sin \alpha = \frac{3}{5}, 0 < \alpha < \frac{\pi}{2}$, and $\sin \beta = \frac{12}{13}, \frac{\pi}{2} < \beta < \pi$.

36. $\tan \alpha = \frac{4}{3}, \pi < \alpha < \frac{3\pi}{2}$, and $\tan \beta = \frac{5}{12}, 0 < \beta < \frac{\pi}{2}$.

37. $\tan \alpha = -3, \frac{\pi}{2} < \alpha < \pi$, and $\cot \beta = -3, \frac{3\pi}{2} < \beta < 2\pi$.

38. $\sin \alpha = -\frac{1}{3}, \pi < \alpha < \frac{3\pi}{2}$, and $\cos \beta = -\frac{1}{3}, \pi < \beta < \frac{3\pi}{2}$.

In Exercises 39–42, use double- and half-angle formulas to find the exact value of each expression.

39. $\cos^2 15° - \sin^2 15°$

40. $\dfrac{2\tan\frac{5\pi}{12}}{1 - \tan^2\frac{5\pi}{12}}$

41. $\sin 22.5°$

42. $\tan\frac{\pi}{12}$

5.4

In Exercises 43–44, express each product as a sum or difference.

43. $\sin 6x \sin 4x$

44. $\sin 7x \cos 3x$

In Exercises 45–46, express each sum or difference as a product. If possible, find this product's exact value.

45. $\sin 2x - \sin 4x$

46. $\cos 75° + \cos 15°$

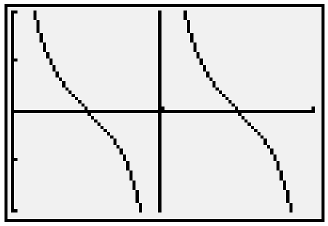
In Exercises 47–48, verify each identity.

47. $\frac{\cos 3x + \cos 5x}{\cos 3x - \cos 5x} = \cot x \cot 4x$

48. $\frac{\sin 2x + \sin 6x}{\sin 2x - \sin 6x} = -\tan 4x \cot 2x$

49. The graph with the given equation is shown in a $\left[0, 2\pi, \frac{\pi}{2}\right]$ by $[-2, 2, 1]$ viewing rectangle.

$$y = \frac{\cos 3x + \cos x}{\sin 3x - \sin x}$$



a. Describe the graph using another equation.

b. Verify that the two equations are equivalent.

5.5

In Exercises 50–53, find all solutions of each equation.

50. $\cos x = -\frac{1}{2}$

51. $\sin x = \frac{\sqrt{2}}{2}$

52. $2 \sin x + 1 = 0$

53. $\sqrt{3} \tan x - 1 = 0$

In Exercises 54–67, solve each equation on the interval $[0, 2\pi)$. Use exact values where possible or give approximate solutions correct to four decimal places.

54. $\cos 2x = -1$

55. $\sin 3x = 1$

56. $\tan\frac{x}{2} = -1$

57. $\tan x = 2 \cos x \tan x$

58. $\cos^2 x - 2 \cos x = 3$

59. $2 \cos^2 x - \sin x = 1$

60. $4 \sin^2 x = 1$

61. $\cos 2x - \sin x = 1$

62. $\sin 2x = \sqrt{3} \sin x$

63. $\sin x = \tan x$

64. $\sin x = -0.6031$

65. $5 \cos^2 x - 3 = 0$

66. $\sec^2 x = 4 \tan x - 2$

67. $2 \sin^2 x + \sin x - 2 = 0$

68. A ball on a spring is pulled 6 inches below its rest position and then released. After t seconds, the ball's distance, d, in inches from its rest position is given by

$$d = -6 \cos\frac{\pi}{2}t.$$

Find all values of t for which the ball is 3 inches below its rest position.

69. You are playing catch with a friend located 100 feet away. If you throw the ball with an initial velocity of $v_0 = 90$ feet per second, at what angle of elevation, θ, to the nearest degree should you direct your throw so that it can be caught easily? Use the formula

$$d = \frac{v_0^2}{16} \sin \theta \cos \theta.$$

CHAPTER Test Prep VIDEOS

Chapter 5 Test

Use the following conditions to solve Exercises 1–4:

$$\sin\alpha = \frac{4}{5},\ \frac{\pi}{2} < \alpha < \pi$$
$$\cos\beta = \frac{5}{13},\ 0 < \beta < \frac{\pi}{2}$$

Find the exact value of each of the following.

1. $\cos(\alpha + \beta)$

2. $\tan(\alpha - \beta)$

3. $\sin 2\alpha$

4. $\cos\frac{\beta}{2}$

5. Use $105° = 135° - 30°$ to find the exact value of $\sin 105°$.

In Exercises 6–11, verify each identity.

6. $\cos x \csc x = \cot x$

7. $\dfrac{\sec x}{\cot x + \tan x} = \sin x$

8. $1 - \dfrac{\cos^2 x}{1 + \sin x} = \sin x$

9. $\cos\left(\theta + \dfrac{\pi}{2}\right) = -\sin\theta$

10. $\dfrac{\sin(\alpha - \beta)}{\sin\alpha\cos\beta} = 1 - \cot\alpha\tan\beta$

11. $\sin t\cos t(\tan t + \cot t) = 1$

In Exercises 12–18, solve each equation on the interval $[0, 2\pi)$. Use exact values where possible or give approximate solutions correct to four decimal places.

12. $\sin 3x = -\frac{1}{2}$

13. $\sin 2x + \cos x = 0$

14. $2\cos^2 x - 3\cos x + 1 = 0$

15. $2\sin^2 x + \cos x = 1$

16. $\cos x = -0.8092$

17. $\tan x \sec x = 3\tan x$

18. $\tan^2 x - 3\tan x - 2 = 0$

Cumulative Review Exercises (Chapters P–5)

Solve each equation or inequality in Exercises 1–5.

1. $x^3 + x^2 - x + 15 = 0$

2. $11^{x-1} = 125$

3. $x^2 + 2x - 8 > 0$

4. $\cos 2x + 3 = 5\cos x, \quad 0 \le x < 2\pi$

5. $\tan x + \sec^2 x = 3, \quad 0 \le x < 2\pi$

In Exercises 6–11, graph each equation.

6. $y = \sqrt{x+2} - 1$; Use transformations of the graph of $y = \sqrt{x}$.

7. $(x - 1)^2 + (y + 2)^2 = 9$

8. $y + 2 = \frac{1}{3}(x - 1)$

9. $y = 3\cos 2x, \quad -2\pi \le x \le 2\pi$

10. $y = 2\sin\frac{x}{2} + 1, \quad -2\pi \le x \le 2\pi$

11. $f(x) = (x - 1)^2(x - 3)$

12. If $f(x) = x^2 + 3x - 1$, find $\dfrac{f(a+h) - f(a)}{h}$.

13. Find the exact value of $\sin 225°$.

14. Verify the identity: $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$.

15. Convert $320°$ to radians.

16. How long would it take for any amount of money, compounded continuously at 5.75% per year, to triple? Round to the nearest tenth of a year.

17. If $f(x) = \dfrac{2x+1}{x-3}$, find $f^{-1}(x)$.

18. If C is a right angle in triangle ABC with $A = 23°$ and $a = 12$, solve the triangle.

19. A formula for calculating an infant's dosage for medication is

$$\text{Infant's dose} = \frac{\text{age of infant in months}}{150} \times \text{adult dose}.$$

If a 12-month-old infant is to receive 8.5 mg of medication, find the equivalent adult dose to the nearest milligram.

20. From a point on the ground 12 feet from the base of a flagpole, the angle of elevation to the top of the pole is 53°. Approximate the height of the flagpole to the nearest tenth of a foot.