Critical Thinking Exercises

Make Sense? *In Exercises 89–92, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

89. I'm working with a polar equation that failed the symmetry test with respect to $\theta = \frac{\pi}{2}$, so my graph will not have this kind of symmetry.

90. The graph of my limaçon exhibits none of the three kinds of symmetry discussed in this section.

91. There are no points on my graph of $r^2 = 9\cos 2\theta$ for which $\frac{\pi}{4} < \theta < \frac{3\pi}{4}$.

92. I'm graphing a polar equation in which for every value of θ there is exactly one corresponding value of r, yet my polar coordinate graph fails the vertical line for functions.

In Exercises 93–94, graph r_1 and r_2 in the same polar coordinate system. What is the relationship between the two graphs?

93. $r_1 = 4\cos 2\theta,\ r_2 = 4\cos 2\left(\theta - \frac{\pi}{4}\right)$

94. $r_1 = 2\sin 3\theta,\ r_2 = 2\sin 3\left(\theta + \frac{\pi}{6}\right)$

95. Describe a test for symmetry with respect to the line $\theta = \frac{\pi}{2}$ in which r is not replaced.

Preview Exercises

Exercises 96–98 will help you prepare for the material covered in the next section. Refer to Section 2.1 if you need to review the basics of complex numbers. In each exercise, perform the indicated operation and write the result in the standard form $a + bi$.

96. $(1 + i)(2 + 2i)$

97. $\left(-1 + i\sqrt{3}\right)\left(-1 + i\sqrt{3}\right)\left(-1 + i\sqrt{3}\right)$

98. $\frac{2 + 2i}{1 + i}$

Chapter 6 Mid-Chapter Check Point

What You Know: We learned to solve oblique triangles using the Laws of Sines $\left(\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}\right)$ and Cosines $(a^2 = b^2 + c^2 - 2bc\cos A)$. We applied the Law of Sines to SAA, ASA, and SSA (the ambiguous case) triangles. We applied the Law of Cosines to SAS and SSS triangles. We found areas of SAS triangles $\left(\text{area} = \frac{1}{2}bc\sin A\right)$ and SSS triangles (Heron's formula: area $= \sqrt{s(s-a)(s-b)(s-c)}$, s is $\frac{1}{2}$ the perimeter). We used the polar coordinate system to plot points and represented them in multiple ways. We used the relations between polar and rectangular coordinates

$$x = r\cos\theta,\ y = r\sin\theta,\ x^2 + y^2 = r^2,\ \tan\theta = \frac{y}{x}$$

to convert points and equations from one coordinate system to the other. Finally, we used point plotting and symmetry to graph polar equations.

In Exercises 1–6, solve each triangle. Round lengths to the nearest tenth and angle measures to the nearest degree. If no triangle exists, state "no triangle." If two triangles exist, solve each triangle.

1. $A = 32°, B = 41°, a = 20$

2. $A = 42°, a = 63, b = 57$

3. $A = 65°, a = 6, b = 7$

4. $B = 110°, a = 10, c = 16$

5. $C = 42°, a = 16, c = 13$

6. $a = 5.0, b = 7.2, c = 10.1$

In Exercises 7–8, find the area of the triangle having the given measurements. Round to the nearest square unit.

7. $C = 36°$, $a = 5$ feet, $b = 7$ feet

8. $a = 7$ meters, $b = 9$ meters, $c = 12$ meters

9. Two trains leave a station on different tracks that make an angle of 110° with the station as vertex. The first train travels at an average rate of 50 miles per hour and the second train travels at an average rate of 40 miles per hour. How far apart, to the nearest tenth of a mile, are the trains after 2 hours?

10. Two fire-lookout stations are 16 miles apart, with station B directly east of station A. Both stations spot a fire on a mountain to the south. The bearing from station A to the fire is S56°E. The bearing from station B to the fire is S23°W. How far, to the nearest tenth of a mile, is the fire from station A?

11. A tree that is perpendicular to the ground sits on a straight line between two people located 420 feet apart. The angles of elevation from each person to the top of the tree measure 50° and 66°, respectively. How tall, to the nearest tenth of a foot, is the tree?

In Exercises 12–15, convert the given coordinates to the indicated ordered pair.

12. $\left(-3, \frac{5\pi}{4}\right)$ to (x, y)

13. $\left(6, -\frac{\pi}{2}\right)$ to (x, y)

14. $\left(2, -2\sqrt{3}\right)$ to (r, θ)

15. $(-6, 0)$ to (r, θ)

In Exercises 16–17, plot each point in polar coordinates. Then find another representation (r, θ) of this point in which:

a. $r > 0,\quad 2\pi < \theta < 4\pi$.

b. $r < 0,\quad 0 < \theta < 2\pi$.

c. $r > 0,\ -2\pi < \theta < 0$.

16. $\left(4, \frac{3\pi}{4}\right)$

17. $\left(\frac{5}{2}, \frac{\pi}{2}\right)$

In Exercises 18–20, convert each rectangular equation to a polar equation that expresses r in terms of θ.

18. $5x - y = 7$

19. $y = -7$

20. $(x + 1)^2 + y^2 = 1$

In Exercises 21–25, convert each polar equation to a rectangular equation. Then use your knowledge of the rectangular equation to graph the polar equation in a polar coordinate system.

21. $r = 6$

22. $\theta = \frac{\pi}{3}$

23. $r = -3 \csc \theta$

24. $r = -10 \cos \theta$

25. $r = 4 \sin \theta \sec^2 \theta$

In Exercises 26–27, test for symmetry with respect to

a. *the polar axis.* **b.** *the line* $\theta = \frac{\pi}{2}$. **c.** *the pole.*

26. $r = 1 - 4 \cos \theta$

27. $r^2 = 4 \cos 2\theta$

In Exercises 28–32, graph each polar equation. Be sure to test for symmetry.

28. $r = -4 \sin \theta$

29. $r = 2 - 2 \cos \theta$

30. $r = 2 - 4 \cos \theta$

31. $r = 2 \sin 3\theta$

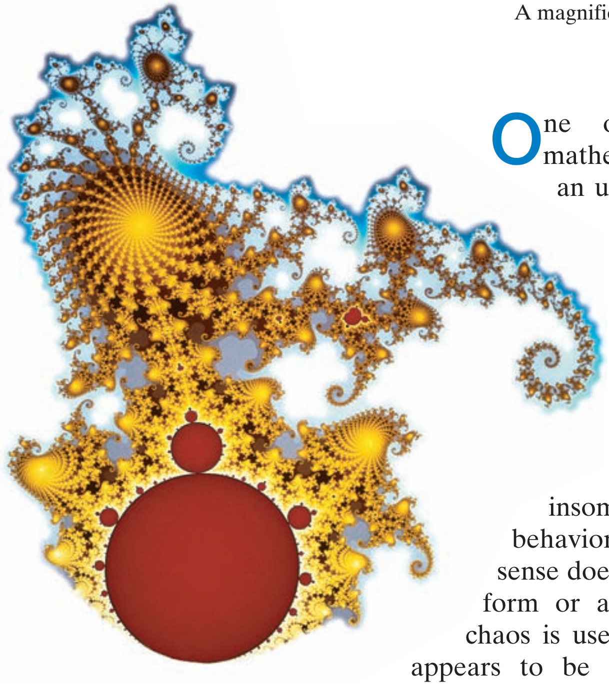
32. $r^2 = 16 \sin 2\theta$

Section 6.5

Complex Numbers in Polar Form; DeMoivre's Theorem

Objectives

1. Plot complex numbers in the complex plane.
2. Find the absolute value of a complex number.
3. Write complex numbers in polar form.
4. Convert a complex number from polar to rectangular form.
5. Find products of complex numbers in polar form.
6. Find quotients of complex numbers in polar form.
7. Find powers of complex numbers in polar form.
8. Find roots of complex numbers in polar form.



A magnification of the Mandelbrot set

One of the new frontiers of mathematics suggests that there is an underlying order in things that appear to be random, such as the hiss and crackle of background noises as you tune a radio. Irregularities in the heartbeat, some of them severe enough to cause a heart attack, or irregularities in our sleeping patterns, such as insomnia, are examples of chaotic behavior. Chaos in the mathematical sense does not mean a complete lack of form or arrangement. In mathematics, chaos is used to describe something that appears to be random but is not actually random. The patterns of chaos appear in images like the one shown above, called the Mandelbrot set. Magnified portions of this image yield repetitions of the original structure, as well as new and unexpected patterns. The Mandelbrot set transforms the hidden structure of chaotic events into a source of wonder and inspiration.

The Mandelbrot set is made possible by opening up graphing to include complex numbers in the form $a + bi$, where $i = \sqrt{-1}$. In this section, you will learn how to graph complex numbers and write them in terms of trigonometric functions.

1 Plot complex numbers in the complex plane.

The Complex Plane

We know that a real number can be represented as a point on a number line. By contrast, a complex number $z = a + bi$ is represented as a point (a, b) in a coordinate plane, as shown in **Figure 6.38** at the top of the next page. The horizontal axis of the coordinate plane is called the **real axis**. The vertical axis is called the **imaginary axis**. The coordinate system is called the **complex plane**. Every complex number