## Chapter P Summary, Review, and Test

## Summary: Basic Formulas

## Definition of Absolute Value

$|x|=\left\{\begin{aligned} x & \text { if } x \geq 0 \\ -x & \text { if } x<0\end{aligned}\right.$
Distance between Points $\boldsymbol{a}$ and $\boldsymbol{b}$ on a Number Line
$|a-b|$ or $|b-a|$

## Properties of Real Numbers

Commutative $a+b=b+a$

$$
a b=b a
$$

Associative $\quad(a+b)+c=a+(b+c)$

$$
(a b) c=a(b c)
$$

Distributive
$a(b+c)=a b+a c$
Identity

$$
a+0=a
$$

$$
a \cdot 1=a
$$

Inverse $\quad a+(-a)=0$

$$
a \cdot \frac{1}{a}=1, a \neq 0
$$

## Properties of Exponents

$$
\begin{aligned}
& b^{-n}=\frac{1}{b^{n}}, \quad b^{0}=1, \quad b^{m} \cdot b^{n}=b^{m+n}, \\
& \left(b^{m}\right)^{n}=b^{m n}, \quad \frac{b^{m}}{b^{n}}=b^{m-n}, \quad(a b)^{n}=a^{n} b^{n}, \quad\left(\frac{a}{b}\right)^{n}=\frac{a^{n}}{b^{n}}
\end{aligned}
$$

## Product and Quotient Rules for nth Roots

$$
\sqrt[n]{a b}=\sqrt[n]{a} \cdot \sqrt[n]{b}, \quad \sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}
$$

## Rational Exponents

$$
\begin{aligned}
& a^{\frac{1}{n}}=\sqrt[n]{a}, \quad a^{-\frac{1}{n}}=\frac{1}{a^{\frac{1}{n}}}=\frac{1}{\sqrt[n]{a}} \\
& a^{\frac{m}{n}}=(\sqrt[n]{a})^{m}=\sqrt[n]{a^{m}}, \quad a^{-\frac{m}{n}}=\frac{1}{a^{\frac{m}{n}}}
\end{aligned}
$$

## Special Products

$$
\begin{aligned}
(A+B)(A-B) & =A^{2}-B^{2} \\
(A+B)^{2} & =A^{2}+2 A B+B^{2} \\
(A-B)^{2} & =A^{2}-2 A B+B^{2} \\
(A+B)^{3} & =A^{3}+3 A^{2} B+3 A B^{2}+B^{3} \\
(A-B)^{3} & =A^{3}-3 A^{2} B+3 A B^{2}-B^{3}
\end{aligned}
$$

## Factoring Formulas

$$
\begin{aligned}
A^{2}-B^{2} & =(A+B)(A-B) \\
A^{2}+2 A B+B^{2} & =(A+B)^{2} \\
A^{2}-2 A B+B^{2} & =(A-B)^{2} \\
A^{3}+B^{3} & =(A+B)\left(A^{2}-A B+B^{2}\right) \\
A^{3}-B^{3} & =(A-B)\left(A^{2}+A B+B^{2}\right)
\end{aligned}
$$

## Absolute Value Equations and Inequalities

1. If $c>0$, then $|u|=c$ is equivalent to $u=c$ or $u=-c$.
2. If $c>0$, then $|u|<c$ is equivalent to $-c<u<c$.
3. If $c>0$, then $|u|>c$ is equivalent to $u<-c$ or $u>c$.

## The Quadratic Formula

All quadratic equations

$$
a x^{2}+b x+c=0, \quad a \neq 0
$$

can be solved by the quadratic formula

$$
x=\frac{-b \pm \sqrt{b^{2}-4 a c}}{2 a} .
$$

## Review Exercises

You can use these review exercises, like the review exercises at the end of each chapter, to test your understanding of the chapter's topics. However, you can also use these exercises as a prerequisite test to check your mastery of the fundamental algebra skills needed in this book.

## P.I

In Exercises 1-2, evaluate each algebraic expression for the given value or values of the variable(s).

$$
\begin{aligned}
& \text { 1. } 3+6(x-2)^{3} \text { for } x=4 \\
& \text { 2. } x^{2}-5(x-y) \text { for } x=6 \text { and } y=2
\end{aligned}
$$

3. You are riding along an expressway traveling $x$ miles per hour. The formula

$$
S=0.015 x^{2}+x+10
$$

models the recommended safe distance, $S$, in feet, between your car and other cars on the expressway. What is the recommended safe distance when your speed is 60 miles per hour?
In Exercises 4-7, let $A=\{a, b, c\}, B=\{a, c, d, e\}$, and $C=\{a, d, f, g\}$. Find the indicated set.
4. $A \cap B$
5. $A \cup B$
6. $A \cup C$
7. $C \cap A$
8. Consider the set:

$$
\left\{-17,-\frac{9}{13}, 0,0.75, \sqrt{2}, \pi, \sqrt{81}\right\} .
$$

List all numbers from the set that are a. natural numbers, b. whole numbers, c. integers, d. rational numbers, e. irrational numbers, $\mathbf{f}$. real numbers.

In Exercises 9-11, rewrite each expression without absolute value bars.
9. $|-103|$
10. $|\sqrt{2}-1|$
11. $|3-\sqrt{17}|$
12. Express the distance between the numbers -17 and 4 using absolute value. Then evaluate the absolute value.

In Exercises 13-18, state the name of the property illustrated.
13. $3+17=17+3$
14. $(6 \cdot 3) \cdot 9=6 \cdot(3 \cdot 9)$
15. $\sqrt{3}(\sqrt{5}+\sqrt{3})=\sqrt{15}+3$
16. $(6 \cdot 9) \cdot 2=2 \cdot(6 \cdot 9)$
17. $\sqrt{3}(\sqrt{5}+\sqrt{3})=(\sqrt{5}+\sqrt{3}) \sqrt{3}$
18. $(3 \cdot 7)+(4 \cdot 7)=(4 \cdot 7)+(3 \cdot 7)$

In Exercises 19-22, simplify each algebraic expression.
19. $5(2 x-3)+7 x$
20. $\frac{1}{5}(5 x)+[(3 y)+(-3 y)]-(-x)$
21. $3(4 y-5)-(7 y+2)$
22. $8-2[3-(5 x-1)]$
23. The bar graph shows the percentage of U.S. adults who have been tested for HIV, by age.


Source: National Center for Health Statistics

The data in the graph can be modeled by the formula

$$
P=-0.05 x^{2}+3.6 x-15
$$

where $P$ represents the percentage of U.S. adults tested for HIV at age $x$. According to the formula, what percentage of U.S. adults who are 21 years old have been tested for HIV? Does the model underestimate or overestimate the percent displayed by the bar graph? By how much?

## P. 2

Evaluate each exponential expression in Exercises 24-27.
24. $(-3)^{3}(-2)^{2}$
25. $2^{-4}+4^{-1}$
26. $5^{-3} \cdot 5$
27. $\frac{3^{3}}{3^{6}}$

Simplify each exponential expression in Exercises 28-31.
28. $\left(-2 x^{4} y^{3}\right)^{3}$
29. $\left(-5 x^{3} y^{2}\right)\left(-2 x^{-11} y^{-2}\right)$
30. $\left(2 x^{3}\right)^{-4}$
31. $\frac{7 x^{5} y^{6}}{28 x^{15} y^{-2}}$

In Exercises 32-33, write each number in decimal notation.
32. $3.74 \times 10^{4}$
33. $7.45 \times 10^{-5}$

In Exercises 34-35, write each number in scientific notation.
34. 3,590,000
35. 0.00725

In Exercises 36-37, perform the indicated operation and write the answer in decimal notation.
36. $\left(3 \times 10^{3}\right)\left(1.3 \times 10^{2}\right)$
37. $\frac{6.9 \times 10^{3}}{3 \times 10^{5}}$
38. In this exercise, use $10^{6}$ for one million and $10^{9}$ for one billion to rewrite the number in each statement in scientific notation.
a. According to the Tax Foundation, in 2005, individuals and companies in the United States spent approximately $\$ 257$ billion on tax preparation costs.
b. According to the Internal Revenue Service, in 2005, approximately 175 million tax returns were filed.
39. Use your scientific notation answers from Exercise 38 to answer this question: If the total 2005 tax preparation costs were evenly divided among all tax returns, how much would it cost to prepare each return? Express the answer in decimal notation, rounded to the nearest dollar.

## P. 3

Use the product rule to simplify the expressions in Exercises 40-43. In Exercises 42-43, assume that variables represent nonnegative real numbers.
40. $\sqrt{300}$
41. $\sqrt{12 x^{2}}$
42. $\sqrt{10 x} \cdot \sqrt{2 x}$
43. $\sqrt{r^{3}}$

Use the quotient rule to simplify the expressions in Exercises 44-45.
44. $\sqrt{\frac{121}{4}}$
45. $\frac{\sqrt{96 x^{3}}}{\sqrt{2 x}}$ (Assume that $x>0$.)

In Exercises 46-48, add or subtract terms whenever possible.
46. $7 \sqrt{5}+13 \sqrt{5}$
47. $2 \sqrt{50}+3 \sqrt{8}$
48. $4 \sqrt{72}-2 \sqrt{48}$

In Exercises 49-52, rationalize the denominator.
49. $\frac{30}{\sqrt{5}}$
50. $\frac{\sqrt{2}}{\sqrt{3}}$
51. $\frac{5}{6+\sqrt{3}}$
52. $\frac{14}{\sqrt{7}-\sqrt{5}}$

Evaluate each expression in Exercises 53-56 or indicate that the root is not a real number.
53. $\sqrt[3]{125}$
54. $\sqrt[5]{-32}$
55. $\sqrt[4]{-125}$
56. $\sqrt[4]{(-5)^{4}}$

Simplify the radical expressions in Exercises 57-61.
57. $\sqrt[3]{81}$
58. $\sqrt[3]{y^{5}}$
59. $\sqrt[4]{8} \cdot \sqrt[4]{10}$
60. $4 \sqrt[3]{16}+5 \sqrt[3]{2}$
61. $\frac{\sqrt[4]{32 x^{5}}}{\sqrt[4]{16 x}}$ (Assume that $x>0$.)

In Exercises 62-67, evaluate each expression.
62. $16^{\frac{1}{2}}$
63. $25^{-\frac{1}{2}}$
64. $125^{\frac{1}{3}}$
65. $27^{-\frac{1}{3}}$
66. $64^{\frac{2}{3}}$
67. $27^{-\frac{4}{3}}$

In Exercises 68-70, simplify using properties of exponents.
68. $\left(5 x^{\frac{2}{3}}\right)\left(4 x^{\frac{1}{4}}\right)$
69. $\frac{15 x^{\frac{3}{4}}}{5 x^{\frac{1}{2}}}$
70. $\left(125 x^{6}\right)^{\frac{2}{3}}$
71. Simplify by reducing the index of the radical: $\sqrt[6]{y^{3}}$.

## P. 4

In Exercises 72-73, perform the indicated operations. Write the resulting polynomial in standard form and indicate its degree.
72. $\left(-6 x^{3}+7 x^{2}-9 x+3\right)+\left(14 x^{3}+3 x^{2}-11 x-7\right)$
73. $\left(13 x^{4}-8 x^{3}+2 x^{2}\right)-\left(5 x^{4}-3 x^{3}+2 x^{2}-6\right)$

In Exercises 74-80, find each product.
74. $(3 x-2)\left(4 x^{2}+3 x-5\right)$
75. $(3 x-5)(2 x+1)$
76. $(4 x+5)(4 x-5)$
77. $(2 x+5)^{2}$
78. $(3 x-4)^{2}$
79. $(2 x+1)^{3}$
80. $(5 x-2)^{3}$

In Exercises 81-87, find each product.
81. $(x+7 y)(3 x-5 y)$
82. $(3 x-5 y)^{2}$
83. $\left(3 x^{2}+2 y\right)^{2}$
84. $(7 x+4 y)(7 x-4 y)$
85. $(a-b)\left(a^{2}+a b+b^{2}\right)$
86. $[5 y-(2 x+1)][5 y+(2 x+1)]$
87. $(x+2 y+4)^{2}$

## P. 5

In Exercises 88-104, factor completely, or state that the polynomial is prime.
88. $15 x^{3}+3 x^{2}$
89. $x^{2}-11 x+28$
90. $15 x^{2}-x-2$
91. $64-x^{2}$
92. $x^{2}+16$
93. $3 x^{4}-9 x^{3}-30 x^{2}$
94. $20 x^{7}-36 x^{3}$
95. $x^{3}-3 x^{2}-9 x+27$
96. $16 x^{2}-40 x+25$
97. $x^{4}-16$
98. $y^{3}-8$
99. $x^{3}+64$
100. $3 x^{4}-12 x^{2}$
101. $27 x^{3}-125$
102. $x^{5}-x$
103. $x^{3}+5 x^{2}-2 x-10$
104. $x^{2}+18 x+81-y^{2}$

In Exercises 105-107, factor and simplify each algebraic expression.
105. $16 x^{-\frac{3}{4}}+32 x^{\frac{1}{4}}$
106. $\left(x^{2}-4\right)\left(x^{2}+3\right)^{\frac{1}{2}}-\left(x^{2}-4\right)^{2}\left(x^{2}+3\right)^{\frac{3}{2}}$
107. $12 x^{-\frac{1}{2}}+6 x^{-\frac{3}{2}}$

## P. 6

In Exercises 108-110, simplify each rational expression. Also, list all numbers that must be excluded from the domain.
108. $\frac{x^{3}+2 x^{2}}{x+2}$
109. $\frac{x^{2}+3 x-18}{x^{2}-36}$
110. $\frac{x^{2}+2 x}{x^{2}+4 x+4}$

In Exercises 111-113, multiply or divide as indicated.
111. $\frac{x^{2}+6 x+9}{x^{2}-4} \cdot \frac{x+3}{x-2}$
112. $\frac{6 x+2}{x^{2}-1} \div \frac{3 x^{2}+x}{x-1}$
113. $\frac{x^{2}-5 x-24}{x^{2}-x-12} \div \frac{x^{2}-10 x+16}{x^{2}+x-6}$

In Exercises 114-117, add or subtract as indicated.
114. $\frac{2 x-7}{x^{2}-9}-\frac{x-10}{x^{2}-9}$
115. $\frac{3 x}{x+2}+\frac{x}{x-2}$
116. $\frac{x}{x^{2}-9}+\frac{x-1}{x^{2}-5 x+6}$
117. $\frac{4 x-1}{2 x^{2}+5 x-3}-\frac{x+3}{6 x^{2}+x-2}$

In Exercises 118-120, simplify each expression.
118. $\frac{\frac{1}{x}-\frac{1}{2}}{\frac{1}{3}-\frac{x}{6}}$
119. $\frac{3+\frac{12}{x}}{1-\frac{16}{x^{2}}}$
120. $\frac{3-\frac{1}{x+3}}{3+\frac{1}{x+3}}$
121. $\frac{\sqrt{25-x^{2}}+\frac{x^{2}}{\sqrt{25-x^{2}}}}{25-x^{2}}$

## P. 7

In Exercises 122-135, solve each equation.
122. $1-2(6-x)=3 x+2$
123. $2(x-4)+3(x+5)=2 x-2$
124. $2 x-4(5 x+1)=3 x+17$
125. $\frac{1}{x-1}-\frac{1}{x+1}=\frac{2}{x^{2}-1}$
126. $\frac{4}{x+2}+\frac{2}{x-4}=\frac{30}{x^{2}-2 x-8}$
127. $-4|2 x+1|+12=0$
128. $2 x^{2}-11 x+5=0$
129. $(3 x+5)(x-3)=5$
130. $3 x^{2}-7 x+1=0$
131. $x^{2}-9=0$
132. $(x-3)^{2}-24=0$
133. $\frac{2 x}{x^{2}+6 x+8}=\frac{x}{x+4}-\frac{2}{x+2}$
134. $\sqrt{8-2 x}-x=0$
135. $\sqrt{2 x-3}+x=3$

In Exercises 136-137, solve each formula for the specified variable.
136. $v t+g t^{2}=s$ for $g$
137. $T=\frac{A-P}{P r}$ for $P$

In Exercises 138-139, without solving the given quadratic equation, determine the number and type of solutions.
138. $x^{2}=2 x-19$
139. $9 x^{2}-30 x+25=0$

## P. 8

In Exercises 140-149, use the five-step strategy for solving word problems.
140. The bar graph represents the millions of barrels of oil consumed each day by the countries with the greatest oil consumption.

## Countries with the Greatest Oil Consumption



Source: U.S. Energy Information Administration

Oil consumption in China exceeds Japan's by 0.8 million barrels per day, and oil consumption in the United States exceeds Japan's by 15 million barrels per day. Of the 82 million barrels of oil used by the world every day, the combined consumption for the United States, China, and Japan is 32.3 million barrels. Determine the daily oil consumption, in millions of barrels, for the United States, China, and Japan.
141. The line graph indicates that in $2000,17.5 \%$ of people in the United States spoke a language other than English at home. For the period from 2000 through 2005, this percentage had been increasing by approximately 0.4 per year. If this trend continues, by which year will $25.1 \%$ of people in the United States speak a language other than English at home?


Source: U.S. Census Bureau
142. After a $20 \%$ price reduction, a cordless phone sold for $\$ 48$. What was the phone's price before the reduction?
143. A salesperson earns $\$ 300$ per week plus $5 \%$ commission of sales. How much must be sold to earn $\$ 800$ in a week?
144. The length of a rectangular field is 6 yards less than triple the width. If the perimeter of the field is 340 yards, what are its dimensions?
145. In 2009 , there were 14,100 students at college A, with a projected enrollment increase of 1500 students per year. In the same year, there were 41,700 students at college B, with a projected enrollment decline of 800 students per year. In which year will the colleges have the same enrollment? What will be the enrollment in each college at that time?
146. An architect is allowed 15 square yards of floor space to add a small bedroom to a house. Because of the room's design in relationship to the existing structure, the width of the rectangular floor must be 7 yards less than two times the length. Find the length and width of the rectangular floor that the architect is permitted.
147. A building casts a shadow that is double the length of its height. If the distance from the end of the shadow to the top of the building is 300 meters, how high is the building? Round to the nearest meter.
148. A painting measuring 10 inches by 16 inches is surrounded by a frame of uniform width. If the combined area of the painting and frame is 280 square inches, determine the width of the frame.
149. Club members equally share the cost of $\$ 1500$ to charter a fishing boat. Shortly before the boat is to leave, four people decide not to go due to rough seas. As a result, the cost per person is increased by $\$ 100$. How many people originally intended to go on the fishing trip?

## P. 9

In Exercises 150-152, express each interval in set-builder notation and graph the interval on a number line.
150. $[-3,5)$
151. $(-2, \infty)$
152. $(-\infty, 0]$

In Exercises 153-156, use graphs to find each set.
153. $(-2,1] \cap[-1,3)$
154. $(-2,1] \cup[-1,3)$
155. $[1,3) \cap(0,4)$
156. $[1,3) \cup(0,4)$

In Exercises 157-166, solve each inequality. Use interval notation to express solution sets and graph each solution set on a number line.
157. $-6 x+3 \leq 15$ 158. $6 x-9 \geq-4 x-3$
159. $\frac{x}{3}-\frac{3}{4}-1>\frac{x}{2}$
160. $6 x+5>-2(x-3)-25$
161. $3(2 x-1)-2(x-4) \geq 7+2(3+4 x)$
162. $7<2 x+3 \leq 9$
163. $|2 x+3| \leq 15$
164. $\left|\frac{2 x+6}{3}\right|>2$
165. $|2 x+5|-7 \geq-6$
166. $-4|x+2|+5 \leq-7$
167. A car rental agency rents a certain car for $\$ 40$ per day with unlimited mileage or $\$ 24$ per day plus $\$ 0.20$ per mile. How far can a customer drive this car per day for the $\$ 24$ option to cost no more than the unlimited mileage option?
168. To receive a B in a course, you must have an average of at least $80 \%$ but less than $90 \%$ on five exams. Your grades on the first four exams were $95 \%, 79 \%, 91 \%$, and $86 \%$. What range of grades on the fifth exam will result in a B for the course?

In Exercises 1-18, simplify the given expression or perform the indicated operation (and simplify, if possible), whichever is appropriate.

1. $5\left(2 x^{2}-6 x\right)-\left(4 x^{2}-3 x\right)$
2. $7+2[3(x+1)-2(3 x-1)]$
3. $\{1,2,5\} \cap\{5, a\}$
4. $\{1,2,5\} \cup\{5, a\}$
5. $\frac{30 x^{3} y^{4}}{6 x^{9} y^{-4}}$
6. $\sqrt{6 r} \sqrt{3 r}$ (Assume that $r \geq 0$.)
7. $4 \sqrt{50}-3 \sqrt{18}$
8. $\frac{3}{5+\sqrt{2}}$
9. $\sqrt[3]{16 x^{4}}$
10. $\frac{x^{2}+2 x-3}{x^{2}-3 x+2}$
11. $\frac{5 \times 10^{-6}}{20 \times 10^{-8}}$ (Express the answer in scientific notation.)
12. $(2 x-5)\left(x^{2}-4 x+3\right)$
13. $(5 x+3 y)^{2}$
14. $\frac{2 x+8}{x-3} \div \frac{x^{2}+5 x+4}{x^{2}-9}$
15. $\frac{x}{x+3}+\frac{5}{x-3}$
16. $\frac{2 x+3}{x^{2}-7 x+12}-\frac{2}{x-3}$
17. $\frac{1-\frac{x}{x+2}}{1+\frac{1}{x}}$
18. $\frac{2 x \sqrt{x^{2}+5}-\frac{2 x^{3}}{\sqrt{x^{2}+5}}}{x^{2}+5}$

In Exercises 19-24, factor completely, or state that the polynomial is prime.
19. $x^{2}-9 x+18$
20. $x^{3}+2 x^{2}+3 x+6$
21. $25 x^{2}-9$
22. $36 x^{2}-84 x+49$
23. $y^{3}-125$
24. $x^{2}+10 x+25-9 y^{2}$
25. Factor and simplify:

$$
x(x+3)^{-\frac{3}{5}}+(x+3)^{\frac{2}{5}}
$$

26. List all the rational numbers in this set:

$$
\left\{-7,-\frac{4}{5}, 0,0.25, \sqrt{3}, \sqrt{4}, \frac{22}{7}, \pi\right\} .
$$

In Exercises 27-28, state the name of the property illustrated.
27. $3(2+5)=3(5+2)$
28. $6(7+4)=6 \cdot 7+6 \cdot 4$
29. Express in scientific notation: 0.00076 .
30. Evaluate: $27^{-\frac{5}{3}}$.
31. In 2007, world population was approximately $6.6 \times 10^{9}$. By some projections, world population will double by 2040. Express the population at that time in scientific notation.
32. Big (Lack of) Men on Campus In 2007, 135 women received bachelor's degrees for every 100 men. According to the U.S. Department of Education, that gender imbalance will widen in the coming years, as shown by the bar graph.

Percentage of Bachelor's Degrees Awarded to United States Men and Women


Source: U.S. Department of Education

The data for bachelor's degrees can be described by the following mathematical models:

a. According to the first formula, what percentage of bachelor's degrees were awarded to men in 2003? Does this underestimate or overestimate the actual percent shown by the bar graph? By how much?
b. Use the given formulas to write a new formula with a rational expression that models the ratio of the percentage of bachelor's degrees received by men to the percentage received by women $n$ years after 1989. Name this new mathematical model $R$, for ratio.
c. Use the formula for $R$ to find the projected ratio of bachelor's degrees received by men to degrees received by women in 2014. According to the model, how many women will receive bachelor's degrees for every two men in 2014? How well does this describe the projections shown by the graph?

In Exercises 33-47, solve each equation or inequality. Use interval notation to express solution sets of inequalities and graph these solution sets on a number line.
33. $7(x-2)=4(x+1)-21$
34. $\frac{2 x-3}{4}=\frac{x-4}{2}-\frac{x+1}{4}$
35. $\frac{2}{x-3}-\frac{4}{x+3}=\frac{8}{x^{2}-9}$
36. $2 x^{2}-3 x-2=0$
37. $(3 x-1)^{2}=75$
38. $x(x-2)=4$
39. $\sqrt{x-3}+5=x$
40. $\sqrt{8-2 x}-x=0$
41. $\left|\frac{2}{3} x-6\right|=2$
42. $-3|4 x-7|+15=0$
43. $\frac{2 x}{x^{2}+6 x+8}+\frac{2}{x+2}=\frac{x}{x+4}$
44. $3(x+4) \geq 5 x-12$
45. $\frac{x}{6}+\frac{1}{8} \leq \frac{x}{2}-\frac{3}{4}$
46. $-3 \leq \frac{2 x+5}{3}<6$
47. $|3 x+2| \geq 3$

In Exercises 48-50, solve each formula for the specified variable.
48. $V=\frac{1}{3} l w h$ for $h$
49. $y-y_{1}=m\left(x-x_{1}\right)$ for $x$
50. $R=\frac{a s}{a+s}$ for $a$

Without changes, the graphs show projections for the amount being paid in Social Security benefits and the amount going into the system. All data are expressed in billions of dollars.


Source: 2004 Social Security Trustees Report

Exercises 51-53 are based on the data shown by the graphs.
51. In 2004, the system's income was $\$ 575$ billion, projected to increase at an average rate of $\$ 43$ billion per year. In which year will the system's income be $\$ 1177$ billion?
52. The data for the system's outflow can be modeled by the formula

$$
B=0.07 x^{2}+47.4 x+500
$$

where $B$ represents the amount paid in benefits, in billions of dollars, $x$ years after 2004. According to this model, when will the amount paid in benefits be $\$ 1177$ billion? Round to the nearest year.
53. How well do your answers to Exercises 51 and 52 model the data shown by the graphs?
54. For every one million U.S. residents, the number of movie theaters exceeds the number of drive-in theaters by 16 and the number of video rental stores exceeds the number of drive-in theaters by 64 . Combined, there are 83 drive-in theaters, movie theaters, and video rental stores per one million U.S. residents. How many of each of these establishments are there per one million residents? (Source: U.S. Census Bureau)
55. The costs for two different kinds of heating systems for a small home are given in the following table. After how many years will total costs for solar heating and electric heating be the same? What will be the cost at that time?

| System | Cost to Install | Operating Cost/Year |
| :--- | :---: | :---: |
| Solar | $\$ 29,700$ | $\$ 150$ |
| Electric | $\$ 5000$ | $\$ 1100$ |

56. The length of a rectangular carpet is 4 feet greater than twice its width. If the area is 48 square feet, find the carpet's length and width.
57. A vertical pole is to be supported by a wire that is 26 feet long and anchored 24 feet from the base of the pole. How far up the pole should the wire be attached?
58. After a $60 \%$ reduction, a jacket sold for $\$ 20$. What was the jacket's price before the reduction?
59. A group of people would like to buy a vacation cabin for $\$ 600,000$, sharing the cost equally. If they could find five more people to join them, each person's share would be reduced by $\$ 6000$. How many people are in the group?
60. You are choosing between two telephone plans for local calls. Plan A charges $\$ 25$ per month for unlimited calls. Plan B has a monthly fee of $\$ 13$ with a charge of $\$ 0.06$ per local call. How many local telephone calls in a month make plan A the better deal?
