152. Irrational numbers cannot be negative.

153. The term x has no coefficient.

154. $5 + 3(x - 4) = 8(x - 4) = 8x - 32$

155. $-x - x = -x + (-x) = 0$

156. $x - 0.02(x + 200) = 0.98x - 4$

In Exercises 157–159, insert either < or > in the shaded area between the numbers to make the statement true.

157. $\sqrt{2}$ ▭ 1.5

158. $-\pi$ ▭ -3.5

159. $-\frac{3.14}{2}$ ▭ $-\frac{\pi}{2}$

Preview Exercises

Exercises 160–162 will help you prepare for the material covered in the next section.

160. In parts (a) and (b), complete each statement.

a. $b^4 \cdot b^3 = (b \cdot b \cdot b \cdot b)(b \cdot b \cdot b) = b^?$

b. $b^5 \cdot b^5 = (b \cdot b \cdot b \cdot b \cdot b)(b \cdot b \cdot b \cdot b \cdot b) = b^?$

c. Generalizing from parts (a) and (b), what should be done with the exponents when multiplying exponential expressions with the same base?

161. In parts (a) and (b), complete each statement.

a. $\frac{b^7}{b^3} = \frac{\cancel{b} \cdot \cancel{b} \cdot \cancel{b} \cdot b \cdot b \cdot b \cdot b}{\cancel{b} \cdot \cancel{b} \cdot \cancel{b}} = b^?$

b. $\frac{b^8}{b^2} = \frac{\cancel{b} \cdot \cancel{b} \cdot b \cdot b \cdot b \cdot b \cdot b \cdot b}{\cancel{b} \cdot \cancel{b}} = b^?$

c. Generalizing from parts (a) and (b), what should be done with the exponents when dividing exponential expressions with the same base?

162. If 6.2 is multiplied by 10^3, what does this multiplication do to the decimal point in 6.2?

Section P.2 Exponents and Scientific Notation

Objectives

1. Use properties of exponents.
2. Simplify exponential expressions.
3. Use scientific notation.

Listening to the radio on the way to campus, you hear politicians discussing the problem of the national debt, which exceeds \$9 trillion. They state that it's more than the gross domestic product of China, the world's second-richest nation, and four times greater than the combined net worth of America's 691 billionaires. They make it seem like the national debt is a real problem, but later you realize that you don't really know what a number like 9 trillion means. If the national debt were evenly divided among all citizens of the country, how much would every man, woman, and child have to pay? Is economic doomsday about to arrive?

In this section, you will learn to use exponents to provide a way of putting large and small numbers in perspective. Using this skill, we will explore the meaning of the national debt.

1 Use properties of exponents.

Properties of Exponents

The major properties of exponents are summarized in the box that follows and continues on the next page.

Study Tip

When a negative integer appears as an exponent, switch the position of the base (from numerator to denominator or from denominator to numerator) and make the exponent positive.

Properties of Exponents

Property	Examples
The Negative-Exponent Rule If b is any real number other than 0 and n is a natural number, then $$b^{-n} = \frac{1}{b^n}.$$	• $5^{-3} = \frac{1}{5^3} = \frac{1}{125}$ • $\frac{1}{4^{-2}} = \frac{1}{\frac{1}{4^2}} = 4^2 = 16$

The Zero-Exponent Rule

If b is any real number other than 0,

$$b^0 = 1.$$

- $7^0 = 1$
- $(-5)^0 = 1$
- $-5^0 = -1$

Only 5 is raised to the zero power.

The Product Rule

If b is a real number or algebraic expression, and m and n are integers,

$$b^m \cdot b^n = b^{m+n}.$$

- $2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 32$
- $x^{-3} \cdot x^7 = x^{-3+7} = x^4$

When multiplying exponential expressions with the same base, add the exponents. Use this sum as the exponent of the common base.

The Power Rule

If b is a real number or algebraic expression, and m and n are integers,

$$(b^m)^n = b^{mn}.$$

- $(2^2)^3 = 2^{2\cdot3} = 2^6 = 64$
- $(x^{-3})^4 = x^{-3\cdot4} = x^{-12} = \frac{1}{x^{12}}$

When an exponential expression is raised to a power, multiply the exponents. Place the product of the exponents on the base and remove the parentheses.

Study Tip

$\frac{4^3}{4^5}$ and $\frac{4^5}{4^3}$ represent different numbers:

$$\frac{4^3}{4^5} = 4^{3-5} = 4^{-2} = \frac{1}{4^2} = \frac{1}{16}$$

$$\frac{4^5}{4^3} = 4^{5-3} = 4^2 = 16.$$

The Quotient Rule

If b is a nonzero real number or algebraic expression, and m and n are integers,

$$\frac{b^m}{b^n} = b^{m-n}.$$

- $\frac{2^8}{2^4} = 2^{8-4} = 2^4 = 16$
- $\frac{x^3}{x^7} = x^{3-7} = x^{-4} = \frac{1}{x^4}$

When dividing exponential expressions with the same nonzero base, subtract the exponent in the denominator from the exponent in the numerator. Use this difference as the exponent of the common base.

Products Raised to Powers

If a and b are real numbers or algebraic expressions, and n is an integer,

$$(ab)^n = a^n b^n.$$

- $(-2y)^4 = (-2)^4 y^4 = 16y^4$
- $(-2xy)^3 = (-2)^3 x^3 y^3 = -8x^3y^3$

When a product is raised to a power, raise each factor to that power.

Quotients Raised to Powers

If a and b are real numbers, $b \neq 0$, or algebraic expressions, and n is an integer,

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}.$$

- $\left(\frac{2}{5}\right)^4 = \frac{2^4}{5^4} = \frac{16}{625}$
- $\left(-\frac{3}{x}\right)^3 = \frac{(-3)^3}{x^3} = -\frac{27}{x^3}$

When a quotient is raised to a power, raise the numerator to that power and divide by the denominator to that power.

2 Simplify exponential expressions.

Simplifying Exponential Expressions

Properties of exponents are used to simplify exponential expressions. An exponential expression is **simplified** when

- No parentheses appear.
- No powers are raised to powers.
- Each base occurs only once.
- No negative or zero exponents appear.

Simplifying Exponential Expressions

	Example
1. If necessary, remove parentheses by using $(ab)^n = a^n b^n$ or $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$.	$(xy)^3 = x^3y^3$
2. If necessary, simplify powers to powers by using $(b^m)^n = b^{mn}$.	$(x^4)^3 = x^{4\cdot 3} = x^{12}$
3. If necessary, be sure that each base appears only once by using $b^m \cdot b^n = b^{m+n}$ or $\frac{b^m}{b^n} = b^{m-n}$.	$x^4 \cdot x^3 = x^{4+3} = x^7$
4. If necessary, rewrite exponential expressions with zero powers as 1 $(b^0 = 1)$. Furthermore, write the answer with positive exponents by using $b^{-n} = \frac{1}{b^n}$ or $\frac{1}{b^{-n}} = b^n$.	$\frac{x^5}{x^8} = x^{5-8} = x^{-3} = \frac{1}{x^3}$

The following example shows how to simplify exponential expressions. Throughout the example, assume that no variable in a denominator is equal to zero.

EXAMPLE 1 Simplifying Exponential Expressions

Simplify:

a. $(-3x^4y^5)^3$ **b.** $(-7xy^4)(-2x^5y^6)$ **c.** $\frac{-35x^2y^4}{5x^6y^{-8}}$ **d.** $\left(\frac{4x^2}{y}\right)^{-3}$.

Solution

a. $(-3x^4y^5)^3 = (-3)^3(x^4)^3(y^5)^3$ Raise each factor inside the parentheses to the third power.

$= (-3)^3x^{4\cdot 3}y^{5\cdot 3}$ Multiply the exponents when raising powers to powers.

$= -27x^{12}y^{15}$ $(-3)^3 = (-3)(-3)(-3) = -27$

b. $(-7xy^4)(-2x^5y^6) = (-7)(-2)xx^5y^4y^6$ Group factors with the same base.

$= 14x^{1+5}y^{4+6}$ When multiplying expressions with the same base, add the exponents.

$= 14x^6y^{10}$ Simplify.

c. $\dfrac{-35x^2y^4}{5x^6y^{-8}} = \left(\dfrac{-35}{5}\right)\left(\dfrac{x^2}{x^6}\right)\left(\dfrac{y^4}{y^{-8}}\right)$ Group factors with the same base.

$= -7x^{2-6}y^{4-(-8)}$ When dividing expressions with the same base, subtract the exponents.

$= -7x^{-4}y^{12}$ Simplify. Notice that $4 - (-8) = 4 + 8 = 12$.

$= \dfrac{-7y^{12}}{x^4}$ Write as a fraction and move the base with the negative exponent, x^{-4}, to the other side of the fraction bar and make the negative exponent positive.

d. $\left(\dfrac{4x^2}{y}\right)^{-3} = \dfrac{(4x^2)^{-3}}{y^{-3}}$ Raise the numerator and the denominator to the -3 power.

$= \dfrac{4^{-3}(x^2)^{-3}}{y^{-3}}$ Raise each factor in the numerator to the -3 power.

$= \dfrac{4^{-3}x^{-6}}{y^{-3}}$ Multiply the exponents when raising a power to a power: $(x^2)^{-3} = x^{2(-3)} = x^{-6}$.

$= \dfrac{y^3}{4^3x^6}$ Move each base with a negative exponent to the other side of the fraction bar and make each negative exponent positive.

$= \dfrac{y^3}{64x^6}$ $4^3 = 4 \cdot 4 \cdot 4 = 64$

Check Point 7 Simplify:

a. $(2x^3y^6)^4$ **b.** $(-6x^2y^5)(3xy^3)$ **c.** $\dfrac{100x^{12}y^2}{20x^{16}y^{-4}}$ **d.** $\left(\dfrac{5x}{y^4}\right)^{-2}$.

Study Tip

Try to avoid the following common errors that can occur when simplifying exponential expressions.

Correct	Incorrect	Description of Error
$b^3 \cdot b^4 = b^7$	~~$b^3 \cdot b^4 = b^{12}$~~	The exponents should be added, not multiplied.
$3^2 \cdot 3^4 = 3^6$	~~$3^2 \cdot 3^4 = 9^6$~~	The common base should be retained, not multiplied.
$\dfrac{5^{16}}{5^4} = 5^{12}$	~~$\dfrac{5^{16}}{5^4} = 5^4$~~	The exponents should be subtracted, not divided.
$(4a)^3 = 64a^3$	~~$(4a)^3 = 4a^3$~~	Both factors should be cubed.
$b^{-n} = \dfrac{1}{b^n}$	~~$b^{-n} = -\dfrac{1}{b^n}$~~	Only the exponent should change sign.
$(a+b)^{-1} = \dfrac{1}{a+b}$	~~$(a+b)^{-1} = \dfrac{1}{a} + \dfrac{1}{b}$~~	The exponent applies to the entire expression $a + b$.

3 Use scientific notation.

Table P.3 Names of Large Numbers

10^2	hundred
10^3	thousand
10^6	million
10^9	billion
10^{12}	trillion
10^{15}	quadrillion
10^{18}	quintillion
10^{21}	sextillion
10^{24}	septillion
10^{27}	octillion
10^{30}	nonillion
10^{100}	googol
10^{googol}	googolplex

Scientific Notation

As of December 2007, the national debt of the United States was about \$9.2 trillion. This is the amount of money the government has had to borrow over the years, mostly by selling bonds, because it has spent more than it has collected in taxes. A stack of \$1 bills equaling the national debt would rise to twice the distance from Earth to the moon. Because a trillion is 10^{12} (see **Table P.3**), the national debt can be expressed as

$$9.2 \times 10^{12}.$$

The number 9.2×10^{12} is written in a form called *scientific notation*.

Scientific Notation

A number is written in **scientific notation** when it is expressed in the form

$$a \times 10^n,$$

where the absolute value of a is greater than or equal to 1 and less than 10 $(1 \le |a| < 10)$, and n is an integer.

It is customary to use the multiplication symbol, $\times$, rather than a dot, when writing a number in scientific notation.

Converting from Scientific to Decimal Notation

Here are two examples of numbers in scientific notation:

$$6.4 \times 10^5 \quad \text{means} \quad 640{,}000.$$
$$2.17 \times 10^{-3} \quad \text{means} \quad 0.00217.$$

Do you see that the number with the positive exponent is relatively large and the number with the negative exponent is relatively small?

We can use n, the exponent on the 10 in $a \times 10^n$, to change a number in scientific notation to decimal notation. If n is **positive**, move the decimal point in a to the **right** n places. If n is **negative**, move the decimal point in a to the **left** $|n|$ places.

EXAMPLE 2 Converting from Scientific to Decimal Notation

Write each number in decimal notation:

a. 6.2×10^7 **b.** -6.2×10^7 **c.** 2.019×10^{-3} **d.** -2.019×10^{-3}.

Solution In each case, we use the exponent on the 10 to determine how far to move the decimal point and in which direction. In parts (a) and (b), the exponent is positive, so we move the decimal point to the right. In parts (c) and (d), the exponent is negative, so we move the decimal point to the left.

a. $6.2 \times 10^7 = 62{,}000{,}000$

$n = 7$ — Move the decimal point 7 places to the right.

b. $-6.2 \times 10^7 = -62{,}000{,}000$

$n = 7$ — Move the decimal point 7 places to the right.

c. $2.019 \times 10^{-3} = 0.002019$

$n = -3$ — Move the decimal point $|-3|$ places, or 3 places, to the left.

d. $-2.019 \times 10^{-3} = -0.002019$

$n = -3$ — Move the decimal point $|-3|$ places, or 3 places, to the left.

Check Point 2 Write each number in decimal notation:

a. -2.6×10^9 **b.** 3.017×10^{-6}.

Converting from Decimal to Scientific Notation

To convert from decimal notation to scientific notation, we reverse the procedure of Example 2.

Converting from Decimal to Scientific Notation

Write the number in the form $a \times 10^n$.

- Determine a, the numerical factor. Move the decimal point in the given number to obtain a number whose absolute value is between 1 and 10, including 1.
- Determine n, the exponent on 10^n. The absolute value of n is the number of places the decimal point was moved. The exponent n is positive if the decimal point was moved to the left, negative if the decimal point was moved to the right, and 0 if the decimal point was not moved.

EXAMPLE 3 Converting from Decimal Notation to Scientific Notation

Write each number in scientific notation:

a. 34,970,000,000,000 **b.** −34,970,000,000,000

c. 0.0000000000802 **d.** −0.0000000000802.

Solution

a. $34{,}970{,}000{,}000{,}000 = 3.497 \times 10^{13}$

Move the decimal point to get a number whose absolute value is between 1 and 10.

The decimal point was moved 13 places to the left, so $n = 13$.

b. $-34{,}970{,}000{,}000{,}000 = -3.497 \times 10^{13}$

c. $0.0000000000802 = 8.02 \times 10^{-11}$

Move the decimal point to get a number whose absolute value is between 1 and 10.

The decimal point was moved 11 places to the right, so $n = -11$.

d. $-0.0000000000802 = -8.02 \times 10^{-11}$

Technology

You can use your calculator's EE (enter exponent) or EXP key to convert from decimal to scientific notation. Here is how it's done for 0.0000000000802.

Many Scientific Calculators

Keystrokes

.0000000000802 EE =

Display

8.02 − 11

Many Graphing Calculators

Use the mode setting for scientific notation.

Keystrokes

.0000000000802 ENTER

Display

8.02E − 11

Study Tip

If the absolute value of a number is greater than 10, it will have a positive exponent in scientific notation. If the absolute value of a number is less than 1, it will have a negative exponent in scientific notation.

Check Point 3 Write each number in scientific notation:

a. 5,210,000,000 **b.** −0.00000006893.

EXAMPLE 4 Expressing the U.S. Population in Scientific Notation

As of December 2007, the population of the United States was approximately 303 million. Express the population in scientific notation.

Solution Because a million is 10^6, the 2007 population can be expressed as

$$303 \times 10^6.$$

This factor is not between 1 and 10, so the number is not in scientific notation.

The voice balloon indicates that we need to convert 303 to scientific notation.

$$303 \times 10^6 = (3.03 \times 10^2) \times 10^6 = 3.03 \times 10^{2+6} = 3.03 \times 10^8$$

$303 = 3.03 \times 10^2$

In scientific notation, the population is 3.03×10^8.

Check Point 4 Express 410×10^7 in scientific notation.

Computations with Scientific Notation

Properties of exponents are used to perform computations with numbers that are expressed in scientific notation.

EXAMPLE 5 Computations with Scientific Notation

Perform the indicated computations, writing the answers in scientific notation:

a. $(6.1 \times 10^5)(4 \times 10^{-9})$ **b.** $\dfrac{1.8 \times 10^4}{3 \times 10^{-2}}$.

Solution

a. $(6.1 \times 10^5)(4 \times 10^{-9})$

$= (6.1 \times 4) \times (10^5 \times 10^{-9})$ Regroup factors.

$= 24.4 \times 10^{5+(-9)}$ Add the exponents on 10 and multiply the other parts.

$= 24.4 \times 10^{-4}$ Simplify.

$= (2.44 \times 10^1) \times 10^{-4}$ Convert 24.4 to scientific notation: $24.4 = 2.44 \times 10^1$.

$= 2.44 \times 10^{-3}$ $10^1 \times 10^{-4} = 10^{1+(-4)} = 10^{-3}$

b. $\dfrac{1.8 \times 10^4}{3 \times 10^{-2}} = \left(\dfrac{1.8}{3}\right) \times \left(\dfrac{10^4}{10^{-2}}\right)$ Regroup factors.

$= 0.6 \times 10^{4-(-2)}$ Subtract the exponents on 10 and divide the other parts.

$= 0.6 \times 10^6$ Simplify: $4 - (-2) = 4 + 2 = 6$.

$= (6 \times 10^{-1}) \times 10^6$ Convert 0.6 to scientific notation: $0.6 = 6 \times 10^{-1}$.

$= 6 \times 10^5$ $10^{-1} \times 10^6 = 10^{-1+6} = 10^5$

Technology

$(6.1 \times 10^5)(4 \times 10^{-9})$
On a Calculator:

Many Scientific Calculators

6.1 [EE] 5 [×] 4 [EE] 9 [+/−] [=]

Display

2.44 − 03

Many Graphing Calculators

6.1 [EE] 5 [×] 4 [EE] [(−)] 9 [ENTER]

Display (in scientific notation mode)

2.44E − 3

Check Point 5 Perform the indicated computations, writing the answers in scientific notation:

a. $(7.1 \times 10^5)(5 \times 10^{-7})$ **b.** $\dfrac{1.2 \times 10^6}{3 \times 10^{-3}}$.

Applications: Putting Numbers in Perspective

Due to tax cuts and spending increases, the United States began accumulating large deficits in the 1980s. To finance the deficit, the government had borrowed \$9.2 trillion as of December 2007. The graph in **Figure P.10** shows the national debt increasing over time.

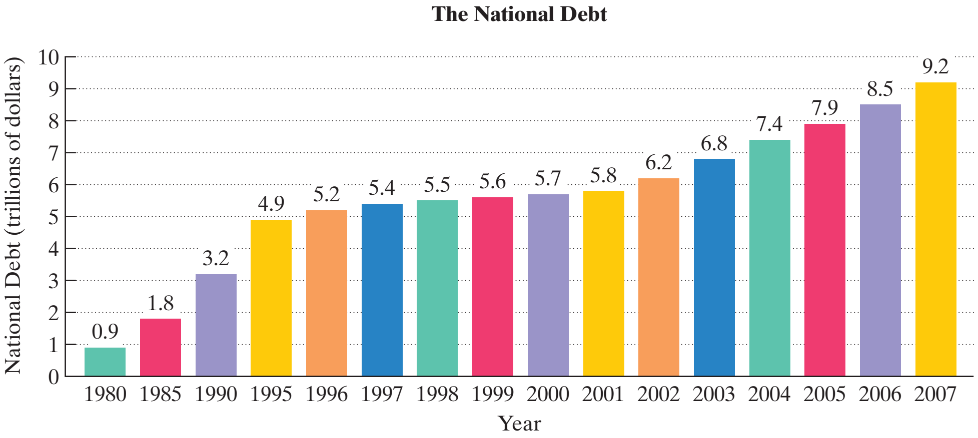


Figure P.10
Source: Office of Management and Budget

Example 6 shows how we can use scientific notation to comprehend the meaning of a number such as 9.2 trillion.

EXAMPLE 6 The National Debt

As of December 2007, the national debt was \$9.2 trillion, or 9.2×10^{12} dollars. At that time, the U.S. population was approximately 303,000,000 (303 million), or 3.03×10^8. If the national debt was evenly divided among every individual in the United States, how much would each citizen have to pay?

Solution The amount each citizen must pay is the total debt, 9.2×10^{12} dollars, divided by the number of citizens, 3.03×10^8.

$$\frac{9.2 \times 10^{12}}{3.03 \times 10^8} = \left(\frac{9.2}{3.03}\right) \times \left(\frac{10^{12}}{10^8}\right)$$
$$\approx 3.04 \times 10^{12-8}$$
$$= 3.04 \times 10^4$$
$$= 30{,}400$$

Every U.S. citizen would have to pay approximately \$30,400 to the federal government to pay off the national debt.

Check Point 6 Pell Grants help low-income undergraduate students pay for college. In 2006, the federal cost of this program was \$13 billion ($\$13 \times 10^9$) and there were 5.1 million (5.1×10^6) grant recipients. How much, to the nearest hundred dollars, was the average grant?

An Application: Black Holes in Space

The concept of a black hole, a region in space where matter appears to vanish, intrigues scientists and nonscientists alike. Scientists theorize that when massive stars run out of nuclear fuel, they begin to collapse under the force of their own gravity. As the star collapses, its density increases. In turn, the force of gravity increases so tremendously that even light cannot escape from the star. Consequently, it appears black.

A mathematical model, called the Schwarzchild formula, describes the critical value to which the radius of a massive body must be reduced for it to become a black hole. This model forms the basis of our next example.

EXAMPLE 7 An Application of Scientific Notation

Use the Schwarzchild formula

$$R_s = \frac{2GM}{c^2}$$

where

R_s = Radius of the star, in meters, that would cause it to become a black hole
M = Mass of the star, in kilograms
G = A constant, called the gravitational constant
$= 6.7 \times 10^{-11} \frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
c = Speed of light
$= 3 \times 10^8$ meters per second

to determine to what length the radius of the sun must be reduced for it to become a black hole. The sun's mass is approximately 2×10^{30} kilograms.

Solution

$$R_s = \frac{2GM}{c^2}$$ Use the given model.

$$= \frac{2 \times 6.7 \times 10^{-11} \times 2 \times 10^{30}}{(3 \times 10^8)^2}$$ Substitute the given values: $G = 6.7 \times 10^{-11}$, $M = 2 \times 10^{30}$, and $c = 3 \times 10^8$.

$$= \frac{(2 \times 6.7 \times 2) \times (10^{-11} \times 10^{30})}{(3 \times 10^8)^2}$$ Rearrange factors in the numerator.

$$= \frac{26.8 \times 10^{-11+30}}{3^2 \times (10^8)^2}$$ Add exponents in the numerator. Raise each factor in the denominator to the power.

$$= \frac{26.8 \times 10^{19}}{9 \times 10^{16}}$$ Multiply powers to powers: $(10^8)^2 = 10^{8 \cdot 2} = 10^{16}$.

$$= \frac{26.8}{9} \times 10^{19-16}$$ When dividing expressions with the same base, subtract the exponents.

$$\approx 2.978 \times 10^3$$ Simplify.

$$= 2978$$

Although the sun is not massive enough to become a black hole (its radius is approximately 700,000 kilometers), the Schwarzchild model theoretically indicates that if the sun's radius were reduced to approximately 2978 meters, that is, about $\frac{1}{235{,}000}$ its present size, it would become a black hole.

Check Point 7 The speed of blood, S, in centimeters per second, located r centimeters from the central axis of an artery is modeled by

$$S = (1.76 \times 10^5)[(1.44 \times 10^{-2}) - r^2].$$

Find the speed of blood at the central axis of this artery.

Exercise Set P.2

Practice Exercises

Evaluate each exponential expression in Exercises 1–22.

1. $5^2 \cdot 2$
2. $6^2 \cdot 2$
3. $(-2)^6$
4. $(-2)^4$
5. -2^6
6. -2^4
7. $(-3)^0$
8. $(-9)^0$
9. -3^0
10. -9^0
11. 4^{-3}
12. 2^{-6}
13. $2^2 \cdot 2^3$
14. $3^3 \cdot 3^2$
15. $(2^2)^3$
16. $(3^3)^2$
17. $\frac{2^8}{2^4}$
18. $\frac{3^8}{3^4}$
19. $3^{-3} \cdot 3$
20. $2^{-3} \cdot 2$
21. $\frac{2^3}{2^7}$
22. $\frac{3^4}{3^7}$

Simplify each exponential expression in Exercises 23–64.

23. $x^{-2}y$
24. xy^{-3}
25. x^0y^5
26. x^7y^0
27. $x^3 \cdot x^7$
28. $x^{11} \cdot x^5$
29. $x^{-5} \cdot x^{10}$
30. $x^{-6} \cdot x^{12}$
31. $(x^3)^7$
32. $(x^{11})^5$
33. $(x^{-5})^3$
34. $(x^{-6})^4$
35. $\frac{x^{14}}{x^7}$
36. $\frac{x^{30}}{x^{10}}$
37. $\frac{x^{14}}{x^{-7}}$
38. $\frac{x^{30}}{x^{-10}}$
39. $(8x^3)^2$
40. $(6x^4)^2$
41. $\left(-\frac{4}{x}\right)^3$
42. $\left(-\frac{6}{y}\right)^3$
43. $(-3x^2y^5)^2$
44. $(-3x^4y^6)^3$
45. $(3x^4)(2x^7)$
46. $(11x^5)(9x^{12})$
47. $(-9x^3y)(-2x^6y^4)$
48. $(-5x^4y)(-6x^7y^{11})$
49. $\frac{8x^{20}}{2x^4}$
50. $\frac{20x^{24}}{10x^6}$
51. $\frac{25a^{13}b^4}{-5a^2b^3}$
52. $\frac{35a^{14}b^6}{-7a^7b^3}$
53. $\frac{14b^7}{7b^{14}}$
54. $\frac{20b^{10}}{10b^{20}}$
55. $(4x^3)^{-2}$
56. $(10x^2)^{-3}$
57. $\frac{24x^3y^5}{32x^7y^{-9}}$
58. $\frac{10x^4y^9}{30x^{12}y^{-3}}$
59. $\left(\frac{5x^3}{y}\right)^{-2}$
60. $\left(\frac{3x^4}{y}\right)^{-3}$
61. $\left(\frac{-15a^4b^2}{5a^{10}b^{-3}}\right)^3$
62. $\left(\frac{-30a^{14}b^8}{10a^{17}b^{-2}}\right)^3$
63. $\left(\frac{3a^{-5}b^2}{12a^3b^{-4}}\right)^0$
64. $\left(\frac{4a^{-5}b^3}{12a^3b^{-5}}\right)^0$

In Exercises 65–76, write each number in decimal notation without the use of exponents.

65. 3.8×10^2
66. 9.2×10^2
67. 6×10^{-4}
68. 7×10^{-5}
69. -7.16×10^6
70. -8.17×10^6
71. 7.9×10^{-1}
72. 6.8×10^{-1}
73. -4.15×10^{-3}
74. -3.14×10^{-3}
75. -6.00001×10^{10}
76. -7.00001×10^{10}

In Exercises 77–86, write each number in scientific notation.

77. 32,000
78. 64,000
79. 638,000,000,000,000,000
80. 579,000,000,000,000,000
81. −5716
82. −3829
83. 0.0027
84. 0.0083
85. −0.00000000504
86. −0.00000000405

In Exercises 87–106, perform the indicated computations. Write the answers in scientific notation. If necessary, round the decimal factor in your scientific notation answer to two decimal places.

87. $(3 \times 10^4)(2.1 \times 10^3)$
88. $(2 \times 10^4)(4.1 \times 10^3)$
89. $(1.6 \times 10^{15})(4 \times 10^{-11})$
90. $(1.4 \times 10^{15})(3 \times 10^{-11})$
91. $(6.1 \times 10^{-8})(2 \times 10^{-4})$
92. $(5.1 \times 10^{-8})(3 \times 10^{-4})$
93. $(4.3 \times 10^8)(6.2 \times 10^4)$
94. $(8.2 \times 10^8)(4.6 \times 10^4)$
95. $\frac{8.4 \times 10^8}{4 \times 10^5}$
96. $\frac{6.9 \times 10^8}{3 \times 10^5}$
97. $\frac{3.6 \times 10^4}{9 \times 10^{-2}}$
98. $\frac{1.2 \times 10^4}{2 \times 10^{-2}}$
99. $\frac{4.8 \times 10^{-2}}{2.4 \times 10^6}$
100. $\frac{7.5 \times 10^{-2}}{2.5 \times 10^6}$
101. $\frac{2.4 \times 10^{-2}}{4.8 \times 10^{-6}}$
102. $\frac{1.5 \times 10^{-2}}{3 \times 10^{-6}}$
103. $\frac{480{,}000{,}000{,}000}{0.00012}$
104. $\frac{282{,}000{,}000{,}000}{0.00141}$
105. $\frac{0.00072 \times 0.003}{0.00024}$
106. $\frac{66{,}000 \times 0.001}{0.003 \times 0.002}$

Practice Plus

In Exercises 107–114, simplify each exponential expression. Assume that variables represent nonzero real numbers.

107. $\dfrac{(x^{-2}y)^{-3}}{(x^2y^{-1})^3}$ **108.** $\dfrac{(xy^{-2})^{-2}}{(x^{-2}y)^{-3}}$

109. $(2x^{-3}yz^{-6})(2x)^{-5}$ **110.** $(3x^{-4}yz^{-7})(3x)^{-3}$

111. $\left(\dfrac{x^3y^4z^5}{x^{-3}y^{-4}z^{-5}}\right)^{-2}$ **112.** $\left(\dfrac{x^4y^5z^6}{x^{-4}y^{-5}z^{-6}}\right)^{-4}$

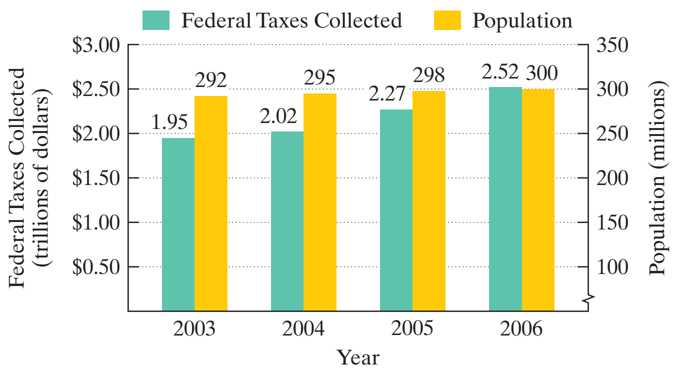
113. $\dfrac{(2^{-1}x^{-2}y^{-1})^{-2}(2x^{-4}y^3)^{-2}(16x^{-3}y^3)^0}{(2x^{-3}y^{-5})^2}$

114. $\dfrac{(2^{-1}x^{-3}y^{-1})^{-2}(2x^{-6}y^4)^{-2}(9x^3y^{-3})^0}{(2x^{-4}y^{-6})^2}$

Application Exercises

The bar graph shows the total amount Americans paid in federal taxes, in trillions of dollars, and the U.S. population, in millions, from 2003 through 2006. Exercises 115–116 are based on the numbers displayed by the graph.

Federal Taxes and the United States Population



Sources: Internal Revenue Service and U.S. Census Bureau

115. a. In 2006, the United States government collected \$2.52 trillion in taxes. Express this number in scientific notation.

b. In 2006, the population of the United States was approximately 300 million. Express this number in scientific notation.

c. Use your scientific notation answers from parts (a) and (b) to answer this question: If the total 2006 tax collections were evenly divided among all Americans, how much would each citizen pay? Express the answer in decimal notation.

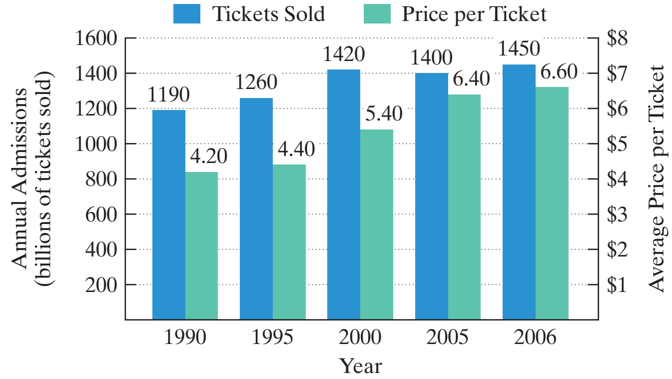
116. a. In 2005, the United States government collected \$2.27 trillion in taxes. Express this number in scientific notation.

b. In 2005, the population of the United States was approximately 298 million. Express this number in scientific notation.

c. Use your scientific notation answers from parts (a) and (b) to answer this question: If the total 2005 tax collections were evenly divided among all Americans, how much would each citizen pay? Express the answer in decimal notation, rounded to the nearest dollar.

In the dramatic arts, ours is the era of the movies. As individuals and as a nation, we've grown up with them. Our images of love, war, family, country—even of things that terrify us—owe much to what we've seen on screen. The bar graph quantifies our love for movies by showing the number of tickets sold, in billions, and the average price per ticket for five selected years. Exercises 117–118 are based on the numbers displayed by the graph.

United States Film Admissions and Admission Charges



Source: National Association of Theater Owners

117. Use scientific notation to compute the amount of money that the motion picture industry made from box-office receipts in 2006. Express the answer in scientific notation.

118. Use scientific notation to compute the amount of money that the motion picture industry made from box office receipts in 2005. Express the answer in scientific notation.

119. The mass of one oxygen molecule is 5.3×10^{-23} gram. Find the mass of 20,000 molecules of oxygen. Express the answer in scientific notation.

120. The mass of one hydrogen atom is 1.67×10^{-24} gram. Find the mass of 80,000 hydrogen atoms. Express the answer in scientific notation.

121. In this exercise, use the fact that there are approximately 3.2×10^7 seconds in a year. According to the United States Department of Agriculture, Americans consume 127 chickens per second. How many chickens are eaten per year in the United States? Express the answer in scientific notation.

122. Convert 365 days (one year) to hours, to minutes, and, finally, to seconds, to determine how many seconds there are in a year. Express the answer in scientific notation.

Writing in Mathematics

123. Describe what it means to raise a number to a power. In your description, include a discussion of the difference between -5^2 and $(-5)^2$.

124. Explain the product rule for exponents. Use $2^3 \cdot 2^5$ in your explanation.

125. Explain the power rule for exponents. Use $(3^2)^4$ in your explanation.

126. Explain the quotient rule for exponents. Use $\dfrac{5^8}{5^2}$ in your explanation.

127. Why is $(-3x^2)(2x^{-5})$ not simplified? What must be done to simplify the expression?

128. How do you know if a number is written in scientific notation?

129. Explain how to convert from scientific to decimal notation and give an example.

130. Explain how to convert from decimal to scientific notation and give an example.

Critical Thinking Exercises

Make Sense? *In Exercises 131–134, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

131. There are many exponential expressions that are equal to $36x^{12}$, such as $(6x^6)^2$, $(6x^3)(6x^9)$, $36(x^3)^9$, and $6^2(x^2)^6$.

132. If 5^{-2} is raised to the third power, the result is a number between 0 and 1.

133. The population of Colorado is approximately 4.6×10^{12}.

134. I just finished reading a book that contained approximately 1.04×10^5 words.

In Exercises 135–142, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

135. $4^{-2} < 4^{-3}$

136. $5^{-2} > 2^{-5}$

137. $(-2)^4 = 2^{-4}$

138. $5^2 \cdot 5^{-2} > 2^5 \cdot 2^{-5}$

139. $534.7 = 5.347 \times 10^3$

140. $\dfrac{8 \times 10^{30}}{4 \times 10^{-5}} = 2 \times 10^{25}$

141. $(7 \times 10^5) + (2 \times 10^{-3}) = 9 \times 10^2$

142. $(4 \times 10^3) + (3 \times 10^2) = 4.3 \times 10^3$

143. The mad Dr. Frankenstein has gathered enough bits and pieces (so to speak) for $2^{-1} + 2^{-2}$ of his creature-to-be. Write a fraction that represents the amount of his creature that must still be obtained.

144. If $b^A = MN$, $b^C = M$, and $b^D = N$, what is the relationship among A, C, and D?

145. Our hearts beat approximately 70 times per minute. Express in scientific notation how many times the heart beats over a lifetime of 80 years. Round the decimal factor in your scientific notation answer to two decimal places.

Group Exercise

146. Putting Numbers into Perspective. A large number can be put into perspective by comparing it with another number. For example, we put the \$9.2 trillion national debt (Example 6) and the \$2.52 trillion the government collected in taxes (Exercise 115) by comparing these numbers to the number of U.S. citizens.

For this project, each group member should consult an almanac, a newspaper, or the World Wide Web to find a number greater than one million. Explain to other members of the group the context in which the large number is used. Express the number in scientific notation. Then put the number into perspective by comparing it with another number.

Preview Exercises

Exercises 147–149 will help you prepare for the material covered in the next section.

147. **a.** Find $\sqrt{16} \cdot \sqrt{4}$.
b. Find $\sqrt{16 \cdot 4}$.
c. Based on your answers to parts (a) and (b), what can you conclude?

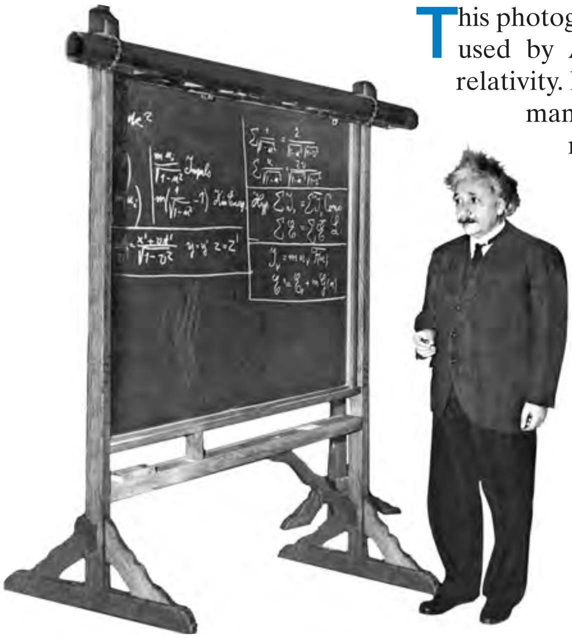
148. **a.** Use a calculator to approximate $\sqrt{300}$ to two decimal places.
b. Use a calculator to approximate $10\sqrt{3}$ to two decimal places.
c. Based on your answers to parts (a) and (b), what can you conclude?

149. **a.** Simplify: $21x + 10x$.
b. Simplify: $21\sqrt{2} + 10\sqrt{2}$.

Section P.3 Radicals and Rational Exponents

Objectives

1. Evaluate square roots.
2. Simplify expressions of the form $\sqrt{a^2}$.
3. Use the product rule to simplify square roots.
4. Use the quotient rule to simplify square roots.
5. Add and subtract square roots.
6. Rationalize denominators.
7. Evaluate and perform operations with higher roots.
8. Understand and use rational exponents.



This photograph shows mathematical models used by Albert Einstein at a lecture on relativity. Notice the radicals that appear in many of the formulas. Among these models, there is one describing how an astronaut in a moving spaceship ages more slowly than friends who remain on Earth. No description of your world can be complete without roots and radicals. In this section, in addition to reviewing the basics of radical expressions and the use of rational exponents to indicate radicals, you will see how radicals model time dilation for a futuristic high-speed trip to a nearby star.