54. I find the hardest part in solving a word problem is writing the equation that models the verbal conditions.

55. After a 35% reduction, a computer's price is \$780, so I determined the original price, x, by solving $x - 0.35 = 780$.

56. When I use the square root property to determine the length of a right triangle's side, I don't even bother to list the negative square root.

57. The perimeter of a plot of land in the shape of a right triangle is 12 miles. If one leg of the triangle exceeds the other leg by 1 mile, find the length of each boundary of the land.

58. The price of a dress is reduced by 40%. When the dress still does not sell, it is reduced by 40% of the reduced price. If the price of the dress after both reductions is \$72, what was the original price?

59. In a film, the actor Charles Coburn plays an elderly "uncle" character criticized for marrying a woman when he is 3 times her age. He wittily replies, "Ah, but in 20 years time I shall only be twice her age." How old is the "uncle" and the woman?

60. Suppose that we agree to pay you 8¢ for every problem in this chapter that you solve correctly and fine you 5¢ for every problem done incorrectly. If at the end of 26 problems we do not owe each other any money, how many problems did you solve correctly?

61. It was wartime when the Ricardos found out Mrs. Ricardo was pregnant. Ricky Ricardo was drafted and made out a will, deciding that \$14,000 in a savings account was to be divided between his wife and his child-to-be. Rather strangely, and certainly with gender bias, Ricky stipulated that if the child were a boy, he would get twice the amount of the mother's portion. If it were a girl, the mother would get twice the amount the girl was to receive. We'll never know what Ricky was thinking of, for (as fate would have it) he did not return from war. Mrs. Ricardo gave birth to twins—a boy and a girl. How was the money divided?

62. A thief steals a number of rare plants from a nursery. On the way out, the thief meets three security guards, one after another. To each security guard, the thief is forced to give one-half the plants that he still has, plus 2 more. Finally, the thief leaves the nursery with 1 lone palm. How many plants were originally stolen?

Group Exercise

63. One of the best ways to learn how to *solve* a word problem in algebra is to *design* word problems of your own. Creating a word problem makes you very aware of precisely how much information is needed to solve the problem. You must also focus on the best way to present information to a reader and on how much information to give. As you write your problem, you gain skills that will help you solve problems created by others.

The group should design five different word problems that can be solved using equations. All of the problems should be on different topics. For example, the group should not have more than one problem on the perimeter of a rectangle. The group should turn in both the problems and their algebraic solutions.

(If you're not sure where to begin, consider the graph for Exercises 5–6 and the data that we did not use regarding attitudes about women in combat.)

Preview Exercises

Exercises 64–66 will help you prepare for the material covered in the next section.

64. Is -1 a solution of $3 - 2x \le 11$?

65. Solve: $-2x - 4 = x + 5$.

66. Solve: $\dfrac{x+3}{4} = \dfrac{x-2}{3} + \dfrac{1}{4}$.

Section P.9 Linear Inequalities and Absolute Value Inequalities

Objectives

1. Use interval notation.
2. Find intersections and unions of intervals.
3. Solve linear inequalities.
4. Solve compound inequalities.
5. Solve absolute value inequalities.

Rent-a-Heap, a car rental company, charges \$125 per week plus \$0.20 per mile to rent one of their cars. Suppose you are limited by how much money you can spend for the week: You can spend at most \$335. If we let x represent the number of miles you drive the heap in a week, we can write an inequality that models the given conditions:

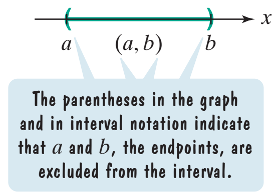
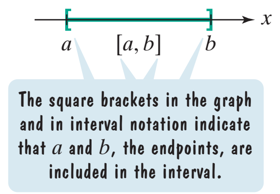
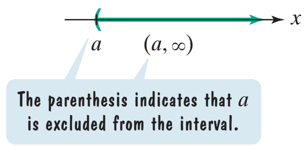
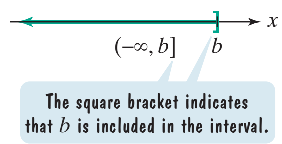
The weekly charge of \$125	plus	the charge of \$0.20 per mile for x miles	must be less than or equal to	\$335.
125	$+$	$0.20x$	$\le$	$335.$

Placing an inequality symbol between a polynomial of degee 1 and a constant results in a *linear inequality in one variable*. In this section, we will study how to solve linear inequalities such as $125 + 0.20x \le 335$. **Solving an inequality** is the process of finding the set of numbers that make the inequality a true statement. These numbers are called the **solutions** of the inequality and we say that they **satisfy** the inequality. The set of all solutions is called the **solution set** of the inequality. Set-builder notation and a new notation, called *interval notation*, are used to represent these solution sets. We begin this section by looking at interval notation.

1 Use interval notation.

Interval Notation

Some sets of real numbers can be represented using **interval notation**. Suppose that a and b are two real numbers such that $a < b$.

Interval Notation	Graph
The **open interval** (a, b) represents the set of real numbers between, but not including, a and b. $(a, b) = \{x \mid a < x < b\}$ x is greater than a $(a < x)$ and x is less than b $(x < b)$.	
The **closed interval** $[a, b]$ represents the set of real numbers between, and including, a and b. $[a, b] = \{x \mid a \le x \le b\}$ x is greater than or equal to a $(a \le x)$ and x is less than or equal to b $(x \le b)$.	
The **infinite interval** (a, ∞) represents the set of real numbers that are greater than a. $(a, \infty) = \{x \mid x > a\}$ The infinity symbol does not represent a real number. It indicates that the interval extends indefinitely to the right.	
The **infinite interval** $(-\infty, b]$ represents the set of real numbers that are less than or equal to b. $(-\infty, b] = \{x \mid x \le b\}$ The negative infinity symbol indicates that the interval extends indefinitely to the left.	

Parentheses and Brackets in Interval Notation

Parentheses indicate endpoints that are not included in an interval. Square brackets indicate endpoints that are included in an interval. Parentheses are always used with ∞ or $-\infty$.

Table P.7 lists nine possible types of intervals used to describe subsets of real numbers.

Table P.7 Intervals on the Real Number Line

Let a and b be real numbers such that $a < b$.

Interval Notation	Set-Builder Notation	Graph
(a, b)	$\{x \mid a < x < b\}$	
$[a, b]$	$\{x \mid a \le x \le b\}$	
$[a, b)$	$\{x \mid a \le x < b\}$	
$(a, b]$	$\{x \mid a < x \le b\}$	
(a, ∞)	$\{x \mid x > a\}$	
$[a, \infty)$	$\{x \mid x \ge a\}$	
$(-\infty, b)$	$\{x \mid x < b\}$	
$(-\infty, b]$	$\{x \mid x \le b\}$	
$(-\infty, \infty)$	$\{x \mid x$ is a real number$\}$ or $\mathbb{R}$ (set of all real numbers)	

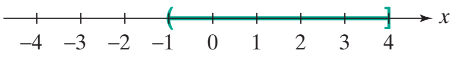
EXAMPLE 1 Using Interval Notation

Express each interval in set-builder notation and graph:

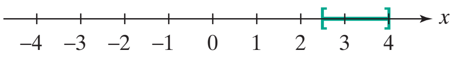
a. $(-1, 4]$ **b.** $[2.5, 4]$ **c.** $(-4, \infty)$.

Solution

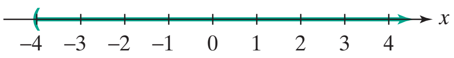
a. $(-1, 4] = \{x \mid -1 < x \le 4\}$



b. $[2.5, 4] = \{x \mid 2.5 \le x \le 4\}$



c. $(-4, \infty) = \{x \mid x > -4\}$



Check Point 1 Express each interval in set-builder notation and graph:

a. $[-2, 5)$ **b.** $[1, 3.5]$ **c.** $(-\infty, -1)$.

2 Find intersections and unions of intervals.

Intersections and Unions of Intervals

In Section P.1, we learned how to find intersections and unions of sets. Recall that $A \cap B$ (A intersection B) is the set of elements common to both set A and set B. By contrast, $A \cup B$ (A union B) is the set of elements in set A or in set B or in both sets.

Because intervals represent sets, it is possible to find their intersections and unions. Graphs are helpful in this process.

Finding Intersections and Unions of Two Intervals

1. Graph each interval on a number line.
2. a. To find the intersection, take the portion of the number line that the two graphs have in common.
   b. To find the union, take the portion of the number line representing the total collection of numbers in the two graphs.

EXAMPLE 2 Finding Intersections and Unions of Intervals

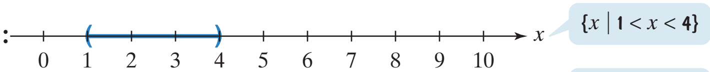
Use graphs to find each set:

a. $(1, 4) \cap [2, 8]$ **b.** $(1, 4) \cup [2, 8]$.

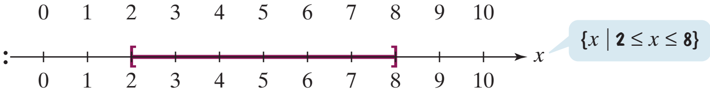
Solution

a. $(1, 4) \cap [2, 8]$, the intersection of the intervals $(1, 4)$ and $[2, 8]$, consists of the numbers that are in both intervals.

Graph of $(1, 4)$:

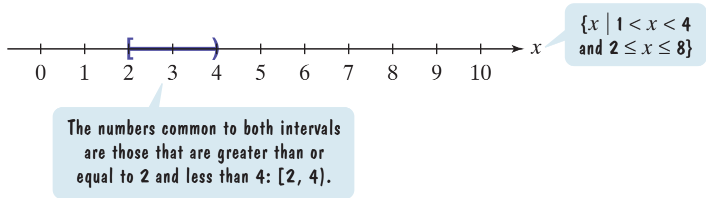


Graph of $[2, 8]$:



To find $(1, 4) \cap [2, 8]$, take the portion of the number line that the two graphs have in common.

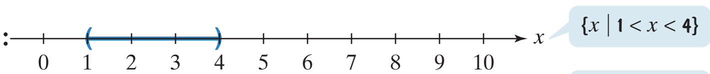
Numbers in both $(1, 4)$ and $[2, 8]$:



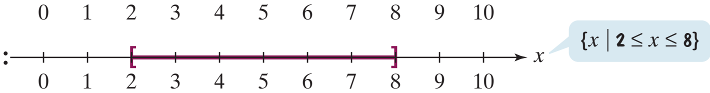
Thus, $(1, 4) \cap [2, 8] = [2, 4)$.

b. $(1, 4) \cup [2, 8]$, the union of the intervals $(1, 4)$ and $[2, 8]$, consists of the numbers that are in either one interval or the other (or both).

Graph of $(1, 4)$:

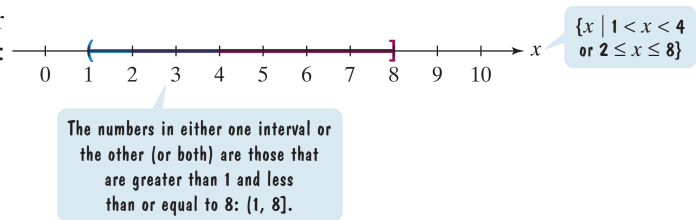


Graph of $[2, 8]$:



To find $(1, 4) \cup [2, 8]$, take the portion of the number line representing the total collection of numbers in the two graphs.

Numbers in either $(1, 4)$ or $[2, 8]$ or both:



Thus, $(1, 4) \cup [2, 8] = (1, 8]$.

Check Point 2 Use graphs to find each set:

a. $[1, 3] \cap (2, 6)$ **b.** $[1, 3] \cup (2, 6)$.

3 Solve linear inequalities.

Solving Linear Inequalities in One Variable

We know that a linear equation in x can be expressed as $ax + b = 0$. A **linear inequality in x** can be written in one of the following forms:

$$ax + b < 0, ax + b \leq 0, ax + b > 0, ax + b \geq 0.$$

In each form, $a \neq 0$.

Back to our question that opened this section: How many miles can you drive your Rent-a-Heap car if you can spend at most \$335? We answer the question by solving

$$0.20x + 125 \leq 335$$

for x. The solution procedure is nearly identical to that for solving

$$0.20x + 125 = 335.$$

Our goal is to get x by itself on the left side. We do this by subtracting 125 from both sides to isolate $0.20x$:

$$0.20x + 125 \leq 335 \qquad \text{This is the given inequality.}$$

$$0.20x + 125 - 125 \leq 335 - 125 \qquad \text{Subtract 125 from both sides.}$$

$$0.02x \leq 210. \qquad \text{Simplify.}$$

Finally, we isolate x from $0.20x$ by dividing both sides of the inequality by 0.20:

$$\frac{0.20x}{0.20} \leq \frac{210}{0.20} \qquad \text{Divide both sides by 0.20.}$$

$$x \leq 1050. \qquad \text{Simplify.}$$

With at most \$335 to spend, you can travel at most 1050 miles.

We started with the inequality $0.20x + 125 \leq 335$ and obtained the inequality $x \leq 1050$ in the final step. These inequalities have the same solution set, namely $\{x | x \leq 1050\}$. Inequalities such as these, with the same solution set, are said to be **equivalent**.

We isolated x from $0.20x$ by dividing both sides of $0.20x \leq 210$ by 0.20, a positive number. Let's see what happens if we divide both sides of an inequality by a negative number. Consider the inequality $10 < 14$. Divide 10 and 14 by -2:

$$\frac{10}{-2} = -5 \quad \text{and} \quad \frac{14}{-2} = -7.$$

Because -5 lies to the right of -7 on the number line, -5 is greater than -7:

$$-5 > -7.$$

Notice that the direction of the inequality symbol is reversed:

$$10 < 14$$
$$-5 > -7.$$

Dividing by -2 changes the direction of the inequality symbol.

In general, **when we multiply or divide both sides of an inequality by a negative number, the direction of the inequality symbol is reversed**. When we reverse the direction of the inequality symbol, we say that we change the *sense* of the inequality.

We can isolate a variable in a linear inequality in the same way we isolate a variable in a linear equation. The properties on the next page are used to create equivalent inequalities.

Study Tip

English phrases such as "at least" and "at most" can be modeled by inequalities.

English Sentence	Inequality
x is at least 5.	$x \geq 5$
x is at most 5.	$x \leq 5$
x is between 5 and 7.	$5 < x < 7$
x is no more than 5.	$x \leq 5$
x is no less than 5.	$x \geq 5$

Properties of Inequalities

Property	The Property in Words	Example
The Addition Property of Inequality If $a < b$, then $a + c < b + c$. If $a < b$, then $a - c < b - c$.	If the same quantity is added to or subtracted from both sides of an inequality, the resulting inequality is equivalent to the original one.	$2x + 3 < 7$ Subtract 3: $2x + 3 - 3 < 7 - 3$. Simplify: $2x < 4$.
The Positive Multiplication Property of Inequality If $a < b$ and c is positive, then $ac < bc$. If $a < b$ and c is positive, then $\frac{a}{c} < \frac{b}{c}$.	If we multiply or divide both sides of an inequality by the same positive quantity, the resulting inequality is equivalent to the original one.	$2x < 4$ Divide by 2: $\frac{2x}{2} < \frac{4}{2}$. Simplify: $x < 2$.
The Negative Multiplication Property of Inequality If $a < b$ and c is negative, then $ac > bc$. If $a < b$ and c is negative, $\frac{a}{c} > \frac{b}{c}$.	If we multiply or divide both sides of an inequality by the same negative quantity and reverse the direction of the inequality symbol, the resulting inequality is equivalent to the original one.	$-4x < 20$ Divide by -4 and change the sense of the inequality: $\frac{-4x}{-4} > \frac{20}{-4}$. Simplify: $x > -5$.

EXAMPLE 3 Solving a Linear Inequality

Solve and graph the solution set on a number line:

$$3 - 2x \le 11.$$

Solution

$$3 - 2x \le 11 \qquad \text{This is the given inequality.}$$

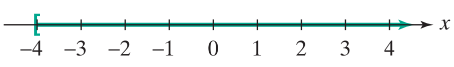
$$3 - 2x - 3 \le 11 - 3 \qquad \text{Subtract 3 from both sides.}$$

$$-2x \le 8 \qquad \text{Simplify.}$$

$$\frac{-2x}{-2} \ge \frac{8}{-2} \qquad \text{Divide both sides by } -2 \text{ and change the sense of the inequality.}$$

$$x \ge -4 \qquad \text{Simplify.}$$

The solution set consists of all real numbers that are greater than or equal to -4, expressed as $\{x | x \ge -4\}$ in set-builder notation. The interval notation for this solution set is $[-4, \infty)$. The graph of the solution set is shown as follows:



Discovery

As a partial check, select one number from the solution set of $3 - 2x \le 11$. Substitute that number into the original inequality. Perform the resulting computations. You should obtain a true statement.

Is it possible to perform a partial check using a number that is not in the solution set? What should happen in this case? Try doing this.

Check Point 3 Solve and graph the solution set on a number line:

$$2 - 3x \le 5.$$

EXAMPLE 4 Solving a Linear Inequality

Solve and graph the solution set on a number line:

$$-2x - 4 > x + 5.$$

Solution

Step 1 Simplify each side. Because each side is already simplified, we can skip this step.

Study Tip

You can solve

$$-2x - 4 > x + 5$$

by isolating x on the right side. Add $2x$ to both sides.

$$\begin{aligned} -2x - 4 + 2x &> x + 5 + 2x \\ -4 &> 3x + 5 \end{aligned}$$

Now subtract 5 from both sides.

$$\begin{aligned} -4 - 5 &> 3x + 5 - 5 \\ -9 &> 3x \end{aligned}$$

Finally, divide both sides by 3.

$$\begin{aligned} \frac{-9}{3} &> \frac{3x}{3} \\ -3 &> x \end{aligned}$$

This last inequality means the same thing as $x < -3$.

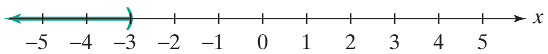
Step 2 Collect variable terms on one side and constant terms on the other side. We will collect the variable terms of $-2x - 4 > x + 5$ on the left and the constant terms on the right.

$-2x - 4 > x + 5$	This is the given inequality.
$-2x - 4 - x > x + 5 - x$	Subtract x from both sides.
$-3x - 4 > 5$	Simplify.
$-3x - 4 + 4 > 5 + 4$	Add 4 to both sides.
$-3x > 9$	Simplify.

Step 3 Isolate the variable and solve. We isolate the variable, x, by dividing both sides by -3. Because we are dividing by a negative number, we must reverse the inequality symbol.

$\frac{-3x}{-3} < \frac{9}{-3}$	Divide both sides by -3 and change the sense of the inequality.
$x < -3$	Simplify.

Step 4 Express the solution set in set-builder or interval notation and graph the set on a number line. The solution set consists of all real numbers that are less than -3, expressed in set-builder notation as $\{x \mid x < -3\}$. The interval notation for this solution set is $(-\infty, -3)$. The graph of the solution set is shown as follows:



Check Point 4 Solve and graph the solution set on a number line: $3x + 1 > 7x - 15$.

If an inequality contains fractions with constants in the denominators, begin by multiplying both sides by the least common denominator. This will clear the inequality of fractions.

EXAMPLE 5 Solving a Linear Inequality Containing Fractions

Solve and graph the solution set on a number line:

$$\frac{x+3}{4} \geq \frac{x-2}{3} + \frac{1}{4}.$$

Solution The denominators are 4, 3, and 4. The least common denominator is 12. We begin by multiplying both sides of the inequality by 12.

$\frac{x+3}{4} \geq \frac{x-2}{3} + \frac{1}{4}$	This is the given inequality.
$12\left(\frac{x+3}{4}\right) \geq 12\left(\frac{x-2}{3} + \frac{1}{4}\right)$	Multiply both sides by 12. Multiplying by a positive number preserves the sense of the inequality.
$\frac{12}{1} \cdot \frac{x+3}{4} \geq \frac{12}{1} \cdot \frac{x-2}{3} + \frac{12}{1} \cdot \frac{1}{4}$	Multiply each term by 12. Use the distributive property on the right side.
$\frac{\overset{3}{\cancel{12}}}{1} \cdot \frac{x+3}{\underset{1}{\cancel{4}}} \geq \frac{\overset{4}{\cancel{12}}}{1} \cdot \frac{x-2}{\underset{1}{\cancel{3}}} + \frac{\overset{3}{\cancel{12}}}{1} \cdot \frac{1}{\underset{1}{\cancel{4}}}$	Divide out common factors in each multiplication.
$3(x+3) \geq 4(x-2) + 3$	The fractions are now cleared.

Now that the fractions have been cleared, we follow the four steps that we used in the previous example.

Step 1 Simplify each side.

$3(x + 3) \geq 4(x - 2) + 3$	This is the inequality with the fractions cleared.
$3x + 9 \geq 4x - 8 + 3$	Use the distributive property.
$3x + 9 \geq 4x - 5$	Simplify.

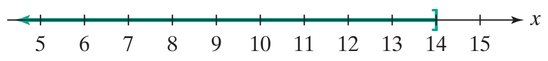
Step 2 Collect variable terms on one side and constant terms on the other side. We will collect variable terms on the left and constant terms on the right.

$3x + 9 - 4x \geq 4x - 5 - 4x$	Subtract 4x from both sides.
$-x + 9 \geq -5$	Simplify.
$-x + 9 - 9 \geq -5 - 9$	Subtract 9 from both sides.
$-x \geq -14$	Simplify.

Step 3 Isolate the variable and solve. To isolate x, we must eliminate the negative sign in front of the x. Because $-x$ means $-1x$, we can do this by multiplying (or dividing) both sides of the inequality by -1. We are multiplying by a negative number. Thus, we must reverse the direction of the inequality symbol.

$(-1)(-x) \leq (-1)(-14)$	Multiply both sides by −1 and change the sense of the in equality.
$x \leq 14$	Simplify.

Step 4 Express the solution set in set-builder or interval notation and graph the set on a number line. The solution set consists of all real numbers that are less than or equal to 14, expressed in set-builder notation as $\{x | x \leq 14\}$. The interval notation for this solution set is $(-\infty, 14]$. The graph of the solution set is shown as follows:



Check Point 5 Solve and graph the solution set on a number line:

$$\frac{x - 4}{2} \geq \frac{x - 2}{3} + \frac{5}{6}.$$

4 Solve compound inequalities.

Solving Compound Inequalities

We now consider two inequalities such as

$$-3 < 2x + 1 \quad \text{and} \quad 2x + 1 \leq 3,$$

expressed as a **compound inequality**

$$-3 < 2x + 1 \leq 3.$$

The word *and* does not appear when the inequality is written in the shorter form, although intersection is implied. The shorter form enables us to solve both inequalities at once. By performing each operation on all three parts of the inequality, our goal is to **isolate x in the middle**.

EXAMPLE 6 Solving a Compound Inequality

Solve and graph the solution set on a number line:

$$-3 < 2x + 1 \leq 3.$$

Solution We would like to isolate x in the middle. We can do this by first subtracting 1 from all three parts of the compound inequality. Then we isolate x from $2x$ by dividing all three parts of the inequality by 2.

$$-3 < 2x + 1 \leq 3 \quad \text{This is the given inequality.}$$

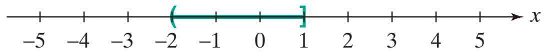
$$-3 - 1 < 2x + 1 - 1 \leq 3 - 1 \quad \text{Subtract 1 from all three parts.}$$

$$-4 < 2x \leq 2 \quad \text{Simplify.}$$

$$\frac{-4}{2} < \frac{2x}{2} \leq \frac{2}{2} \quad \text{Divide each part by 2.}$$

$$-2 < x \leq 1 \quad \text{Simplify.}$$

The solution set consists of all real numbers greater than -2 and less than or equal to 1, represented by $\{x|-2 < x \leq 1\}$ in set-builder notation and $(-2, 1]$ in interval notation. The graph is shown as follows:



Check Point 6 Solve and graph the solution set on a number line: $1 \leq 2x + 3 < 11$.

5 Solve absolute value inequalities.

Solving Inequalities with Absolute Value

We know that $|x|$ describes the distance of x from zero on a real number line. We can use this geometric interpretation to solve an inequality such as

$$|x| < 2.$$

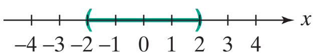


Figure P.19 $|x| < 2$, so $-2 < x < 2$.

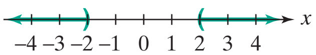


Figure P.20 $|x| > 2$, so $x < -2$ or $x > 2$.

This means that the distance of x from 0 is *less than* 2, as shown in **Figure P.19**. The interval shows values of x that lie less than 2 units from 0. Thus, x can lie between -2 and 2. That is, x is greater than -2 and less than 2. We write $(-2, 2)$ or $\{x|-2 < x < 2\}$.

Some absolute value inequalities use the "greater than" symbol. For example, $|x| > 2$ means that the distance of x from 0 is *greater than* 2, as shown in **Figure P.20**. Thus, x can be less than -2 *or* greater than 2. We write $x < -2$ or $x > 2$. This can be expressed in interval notation as $(-\infty, -2) \cup (2, \infty)$.

These observations suggest the following principles for solving inequalities with absolute value.

Study Tip

In the $|u| < c$ case, we have one compound inequality to solve. In the $|u| > c$ case, we have two separate inequalities to solve.

Solving an Absolute Value Inequality

If u is an algebraic expression and c is a positive number,

1. The solutions of $|u| < c$ are the numbers that satisfy $-c < u < c$.
2. The solutions of $|u| > c$ are the numbers that satisfy $u < -c$ or $u > c$.

These rules are valid if $<$ is replaced by $\leq$ and $>$ is replaced by $\geq$.

EXAMPLE 7 Solving an Absolute Value Inequality

Solve and graph the solution set on a number line: $|x - 4| < 3$.

Solution We rewrite the inequality without absolute value bars.

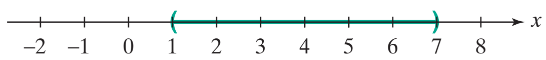
$|u| < c$ means $-c < u < c$.

$$|x - 4| < 3 \quad \text{means} \quad -3 < x - 4 < 3.$$

We solve the compound inequality by adding 4 to all three parts.

$$-3 < x - 4 < 3$$
$$-3 + 4 < x - 4 + 4 < 3 + 4$$
$$1 < x < 7$$

The solution set consists of all real numbers greater than 1 and less than 7, denoted by $\{x|1 < x < 7\}$ or $(1, 7)$. The graph of the solution set is shown as follows:



Check Point 7 Solve and graph the solution set on a number line: $|x - 2| < 5$.

EXAMPLE 8 Solving an Absolute Value Inequality

Solve and graph the solution set on a number line: $-2|3x + 5| + 7 \geq -13$.

Solution

$-2|3x + 5| + 7 \geq -13$ — This is the given inequality.

We need to isolate $|3x + 5|$, the absolute value expression.

$-2|3x + 5| + 7 - 7 \geq -13 - 7$ — Subtract 7 from both sides.

$-2|3x + 5| \geq -20$ — Simplify.

$\dfrac{-2|3x + 5|}{-2} \leq \dfrac{-20}{-2}$ — Divide both sides by -2 and change the sense of the inequality.

$|3x + 5| \leq 10$ — Simplify.

$-10 \leq 3x + 5 \leq 10$ — Rewrite without absolute value bars: $|u| \leq c$ means $-c \leq u \leq c$.

Now we need to isolate x in the middle.

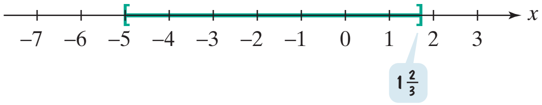
$-10 - 5 \leq 3x + 5 - 5 \leq 10 - 5$ — Subtract 5 from all three parts.

$-15 \leq 3x \leq 5$ — Simplify.

$\dfrac{-15}{3} \leq \dfrac{3x}{3} \leq \dfrac{5}{3}$ — Divide each part by 3.

$-5 \leq x \leq \dfrac{5}{3}$ — Simplify.

The solution set is $\left\{x \middle| -5 \le x \le \frac{5}{3}\right\}$ in set-builder notation and $\left[-5, \frac{5}{3}\right]$ in interval notation. The graph is shown as follows:



Check Point 8 Solve and graph the solution set on a number line: $-3|5x - 2| + 20 \ge -19$.

EXAMPLE 9 Solving an Absolute Value Inequality

Solve and graph the solution set on a number line: $7 < |5 - 2x|$.

Solution We begin by expressing the inequality with the absolute value expression on the left side:

$$|5 - 2x| > 7.$$

$c < |u|$ means the same thing as $|u| > c$. In both cases, the inequality symbol points to c.

We rewrite this inequality without absolute value bars.

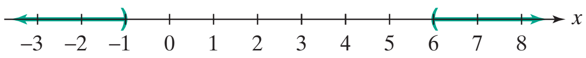
$|u| > c$ means $u < -c$ or $u > c$.

$$|5 - 2x| > 7 \text{ means } 5 - 2x < -7 \text{ or } 5 - 2x > 7.$$

Because $|5 - 2x| > 7$ means $5 - 2x < -7$ or $5 - 2x > 7$, we solve $5 - 2x < -7$ and $5 - 2x > 7$ separately. Then we take the union of their solution sets.

$5 - 2x < -7$	or	$5 - 2x > 7$	These are the inequalities without absolute value bars.
$5 - 5 - 2x < -7 - 5$		$5 - 5 - 2x > 7 - 5$	Subtract 5 from both sides.
$-2x < -12$		$-2x > 2$	Simplify.
$\frac{-2x}{-2} > \frac{-12}{-2}$		$\frac{-2x}{-2} < \frac{2}{-2}$	Divide both sides by -2 and change the sense of each inequality.
$x > 6$		$x < -1$	Simplify.

The solution set consists of all numbers that are less than -1 or greater than 6. The solution set is $\{x | x < -1 \text{ or } x > 6\}$, or, in interval notation $(-\infty, -1) \cup (6, \infty)$. The graph of the solution set is shown as follows:



Study Tip

The graph of the solution set for $|u| > c$ will be divided into two intervals whose union cannot be represented as a single interval. The graph of the solution set for $|u| < c$ will be a single interval. Avoid the common error of rewriting $|u| > c$ as $-c < u > c$.

Check Point 9 Solve and graph the solution set on a number line: $18 < |6 - 3x|$.

Applications

Our next example shows how to use an inequality to select the better deal between two pricing options. We use our strategy for solving word problems, modeling the verbal conditions of the problem with a linear inequality.

EXAMPLE 10 Selecting the Better Deal

Acme Car rental agency charges \$4 a day plus \$0.15 per mile, whereas Interstate rental agency charges \$20 a day and \$0.05 per mile. How many miles must be driven to make the daily cost of an Acme rental a better deal than an Interstate rental?

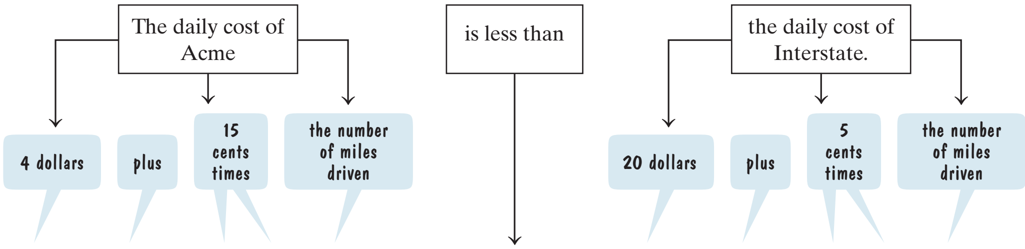
Solution

Step 1 Let *x* represent one of the unknown quantities. We are looking for the number of miles that must be driven in a day to make Acme the better deal. Thus,

let $x =$ the number of miles driven in a day.

Step 2 Represent other unknown quantities in terms of *x*. We are not asked to find another quantity, so we can skip this step.

Step 3 Write an inequality in *x* that models the conditions. Acme is a better deal than Interstate if the daily cost of Acme is less than the daily cost of Interstate.



$$4 + 0.15 \cdot x < 20 + 0.05 \cdot x$$

Step 4 Solve the inequality and answer the question.

$4 + 0.15x < 20 + 0.05x$	This is the inequality that models the verbal conditions.
$4 + 0.15x - 0.05x < 20 + 0.05x - 0.05x$	Subtract 0.05x from both sides.
$4 + 0.1x < 20$	Simplify.
$4 + 0.1x - 4 < 20 - 4$	Subtract 4 from both sides.
$0.1x < 16$	Simplify.
$\frac{0.1x}{0.1} < \frac{16}{0.1}$	Divide both sides by 0.1.
$x < 160$	Simplify.

Thus, driving fewer than 160 miles per day makes Acme the better deal.

Step 5 Check the proposed solution in the original wording of the problem. One way to do this is to take a mileage less than 160 miles per day to see if Acme is the better deal. Suppose that 150 miles are driven in a day.

$$\text{Cost for Acme} = 4 + 0.15(150) = 26.50$$

$$\text{Cost for Interstate} = 20 + 0.05(150) = 27.50$$

Acme has a lower daily cost, making Acme the better deal.

Check Point 10 A car can be rented from Basic Rental for \$260 per week with no extra charge for mileage. Continental charges \$80 per week plus 25 cents for each mile driven to rent the same car. How many miles must be driven in a week to make the rental cost for Basic Rental a better deal than Continental's?

Exercise Set P.9

Practice Exercises

In Exercises 1–14, express each interval in set-builder notation and graph the interval on a number line.

1. $(1, 6]$
2. $(-2, 4]$
3. $[-5, 2)$
4. $[-4, 3)$
5. $[-3, 1]$
6. $[-2, 5]$
7. $(2, \infty)$
8. $(3, \infty)$
9. $[-3, \infty)$
10. $[-5, \infty)$
11. $(-\infty, 3)$
12. $(-\infty, 2)$
13. $(-\infty, 5.5)$
14. $(-\infty, 3.5]$

In Exercises 15–26, use graphs to find each set.

15. $(-3, 0) \cap [-1, 2]$
16. $(-4, 0) \cap [-2, 1]$
17. $(-3, 0) \cup [-1, 2]$
18. $(-4, 0) \cup [-2, 1]$
19. $(-\infty, 5) \cap [1, 8)$
20. $(-\infty, 6) \cap [2, 9)$
21. $(-\infty, 5) \cup [1, 8)$
22. $(-\infty, 6) \cup [2, 9)$
23. $[3, \infty) \cap (6, \infty)$
24. $[2, \infty) \cap (4, \infty)$
25. $[3, \infty) \cup (6, \infty)$
26. $[2, \infty) \cup (4, \infty)$

In all exercises, other than $\varnothing$, use interval notation to express solution sets and graph each solution set on a number line.

In Exercises 27–48, solve each linear inequality.

27. $5x + 11 < 26$
28. $2x + 5 < 17$
29. $3x - 7 \geq 13$
30. $8x - 2 \geq 14$
31. $-9x \geq 36$
32. $-5x \leq 30$
33. $8x - 11 \leq 3x - 13$
34. $18x + 45 \leq 12x - 8$
35. $4(x + 1) + 2 \geq 3x + 6$
36. $8x + 3 > 3(2x + 1) + x + 5$
37. $2x - 11 < -3(x + 2)$
38. $-4(x + 2) > 3x + 20$
39. $1 - (x + 3) \geq 4 - 2x$
40. $5(3 - x) \leq 3x - 1$
41. $\frac{x}{4} - \frac{3}{2} \leq \frac{x}{2} + 1$
42. $\frac{3x}{10} + 1 \geq \frac{1}{5} - \frac{x}{10}$
43. $1 - \frac{x}{2} > 4$
44. $7 - \frac{4}{5}x < \frac{3}{5}$
45. $\frac{x - 4}{6} \geq \frac{x - 2}{9} + \frac{5}{18}$
46. $\frac{4x - 3}{6} + 2 \geq \frac{2x - 1}{12}$
47. $3[3(x + 5) + 8x + 7] + 5[3(x - 6) - 2(3x - 5)] < 2(4x + 3)$
48. $5[3(2 - 3x) - 2(5 - x)] - 6[5(x - 2) - 2(4x - 3)] < 3x + 19$

In Exercises 49–56, solve each compound inequality.

49. $6 < x + 3 < 8$
50. $7 < x + 5 < 11$
51. $-3 \leq x - 2 < 1$
52. $-6 < x - 4 \leq 1$
53. $-11 < 2x - 1 \leq -5$
54. $3 \leq 4x - 3 < 19$
55. $-3 \leq \frac{2}{3}x - 5 < -1$
56. $-6 \leq \frac{1}{2}x - 4 < -3$

In Exercises 57–92, solve each absolute value inequality.

57. $|x| < 3$
58. $|x| < 5$
59. $|x - 1| \leq 2$
60. $|x + 3| \leq 4$
61. $|2x - 6| < 8$
62. $|3x + 5| < 17$
63. $|2(x - 1) + 4| \leq 8$
64. $|3(x - 1) + 2| \leq 20$
65. $\left|\frac{2x + 6}{3}\right| < 2$
66. $\left|\frac{3(x - 1)}{4}\right| < 6$
67. $|x| > 3$
68. $|x| > 5$
69. $|x - 1| \geq 2$
70. $|x + 3| \geq 4$
71. $|3x - 8| > 7$
72. $|5x - 2| > 13$
73. $\left|\frac{2x + 2}{4}\right| \geq 2$
74. $\left|\frac{3x - 3}{9}\right| \geq 1$
75. $\left|3 - \frac{2}{3}x\right| > 5$
76. $\left|3 - \frac{3}{4}x\right| > 9$
77. $3|x - 1| + 2 \geq 8$
78. $5|2x + 1| - 3 \geq 9$
79. $-2|x - 4| \geq -4$
80. $-3|x + 7| \geq -27$
81. $-4|1 - x| < -16$
82. $-2|5 - x| < -6$
83. $3 \leq |2x - 1|$
84. $9 \leq |4x + 7|$
85. $5 > |4 - x|$
86. $2 > |11 - x|$
87. $1 < |2 - 3x|$
88. $4 < |2 - x|$
89. $12 < \left|-2x + \frac{6}{7}\right| + \frac{3}{7}$
90. $1 < \left|x - \frac{11}{3}\right| + \frac{7}{3}$
91. $4 + \left|3 - \frac{x}{3}\right| \geq 9$
92. $\left|2 - \frac{x}{2}\right| - 1 \leq 1$

Practice Plus

In Exercises 93–96, use interval notation to represent all values of x satisfying the given conditions.

93. $y = 1 - (x + 3) + 2x$ and y is at least 4.

94. $y = 2x - 11 + 3(x + 2)$ and y is at most 0.

95. $y = 7 - \left|\frac{x}{2} + 2\right|$ and y is at most 4.

96. $y = 8 - |5x + 3|$ and y is at least 6.

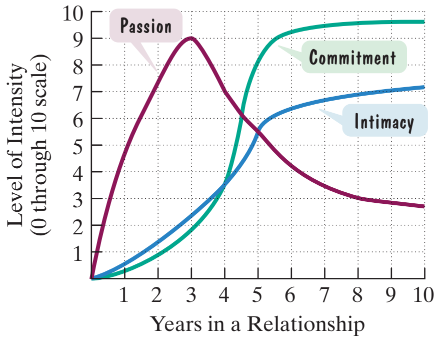
97. When 3 times a number is subtracted from 4, the absolute value of the difference is at least 5. Use interval notation to express the set of all numbers that satisfy this condition.

98. When 4 times a number is subtracted from 5, the absolute value of the difference is at most 13. Use interval notation to express the set of all numbers that satisfy this condition.

Application Exercises

The graphs show that the three components of love, namely passion, intimacy, and commitment, progress differently over time. Passion peaks early in a relationship and then declines. By contrast, intimacy and commitment build gradually. Use the graphs to solve Exercises 99–106.

The Course of Love Over Time



Source: R. J. Sternberg. A Triangular Theory of Love, *Psychological Review*, 93, 119–135.

99. Use interval notation to write an inequality that expresses for which years in a relationship intimacy is greater than commitment.

100. Use interval notation to write an inequality that expresses for which years in a relationship passion is greater than or equal to intimacy.

101. What is the relationship between passion and intimacy on for years $[5, 7)$?

102. What is the relationship between intimacy and commitment for years $[4, 7)$?

103. What is the relationship between passion and commitment for years $(6, 8)$?

104. What is the relationship between passion and commitment for years $(7, 9)$?

105. What is the maximum level of intensity for passion? After how many years in a relationship does this occur?

106. After approximately how many years do levels of intensity for commitment exceed the maximum level of intensity for passion?

107. The percentage, P, of U.S. voters who used electronic voting systems, such as optical scans, in national elections can be modeled by the formula

$$P = 3.1x + 25.8,$$

where x is the number of years after 1994. In which years will more than 63% of U.S. voters use electronic systems?

108. The percentage, P, of U.S. voters who used punch cards or lever machines in national elections can be modeled by the formula

$$P = -2.5x + 63.1,$$

where x is the number of years after 1994. In which years will fewer than 38.1% of U.S. voters use punch cards or lever machines?

109. A basic cellular phone plan costs \$20 per month for 60 calling minutes. Additional time costs \$0.40 per minute. The formula

$$C = 20 + 0.40(x - 60)$$

gives the monthly cost for this plan, C, for x calling minutes, where $x > 60$. How many calling minutes are possible for a monthly cost of at least \$28 and at most \$40?

110. The formula for converting Fahrenheit temperature, F, to Celsius temperature, C, is

$$C = \frac{5}{9}(F - 32).$$

If Celsius temperature ranges from 15° to 35°, inclusive, what is the range for the Fahrenheit temperature? Use interval notation to express this range.

111. If a coin is tossed 100 times, we would expect approximately 50 of the outcomes to be heads. It can be demonstrated that a coin is unfair if h, the number of outcomes that result in heads, satisfies $\left|\frac{h - 50}{5}\right| \geq 1.645$. Describe the number of outcomes that determine an unfair coin that is tossed 100 times.

In Exercises 112–123, use the strategy for solving word problems, modeling the verbal conditions of the problem with a linear inequality.

112. A truck can be rented from Basic Rental for \$50 per day plus \$0.20 per mile. Continental charges \$20 per day plus \$0.50 per mile to rent the same truck. How many miles must be driven in a day to make the rental cost for Basic Rental a better deal than Continental's?

113. You are choosing between two long-distance telephone plans. Plan A has a monthly fee of \$15 with a charge of \$0.08 per minute for all long-distance calls. Plan B has a monthly fee of \$3 with a charge of \$0.12 per minute for all long-distance calls. How many minutes of long-distance calls in a month make plan A the better deal?

114. A city commission has proposed two tax bills. The first bill requires that a homeowner pay \$1800 plus 3% of the assessed home value in taxes. The second bill requires taxes of \$200 plus 8% of the assessed home value. What price range of home assessment would make the first bill a better deal?

115. A local bank charges \$8 per month plus 5¢ per check. The credit union charges \$2 per month plus 8¢ per check. How many checks should be written each month to make the credit union a better deal?

116. A company manufactures and sells blank audiocassette tapes. The weekly fixed cost is \$10,000 and it costs \$0.40 to produce each tape. The selling price is \$2.00 per tape. How many tapes must be produced and sold each week for the company to generate a profit?

117. A company manufactures and sells personalized stationery. The weekly fixed cost is \$3000 and it costs \$3.00 to produce each package of stationery. The selling price is \$5.50 per package. How many packages of stationery must be produced and sold each week for the company to generate a profit?

118. An elevator at a construction site has a maximum capacity of 2800 pounds. If the elevator operator weighs 265 pounds and each cement bag weighs 65 pounds, how many bags of cement can be safely lifted on the elevator in one trip?

119. An elevator at a construction site has a maximum capacity of 3000 pounds. If the elevator operator weighs 245 pounds and each cement bag weighs 95 pounds, how many bags of cement can be safely lifted on the elevator in one trip?

120. To earn an A in a course, you must have a final average of at least 90%. On the first four examinations, you have grades of 86%, 88%, 92%, and 84%. If the final examination counts as two grades, what must you get on the final to earn an A in the course?

121. On two examinations, you have grades of 86 and 88. There is an optional final examination, which counts as one grade. You decide to take the final in order to get a course grade of A, meaning a final average of at least 90.

   a. What must you get on the final to earn an A in the course?

   b. By taking the final, if you do poorly, you might risk the B that you have in the course based on the first two exam grades. If your final average is less than 80, you will lose your B in the course. Describe the grades on the final that will cause this to happen.

122. Parts for an automobile repair cost \$175. The mechanic charges \$34 per hour. If you receive an estimate for at least \$226 and at most \$294 for fixing the car, what is the time interval that the mechanic will be working on the job?

123. The toll to a bridge is \$3.00. A three-month pass costs \$7.50 and reduces the toll to \$0.50. A six-month pass costs \$30 and permits crossing the bridge for no additional fee. How many crossings per three-month period does it take for the three-month pass to be the best deal?

Writing in Mathematics

124. When graphing the solutions of an inequality, what does a parenthesis signify? What does a bracket signify?

125. Describe ways in which solving a linear inequality is similar to solving a linear equation.

126. Describe ways in which solving a linear inequality is different than solving a linear equation.

127. What is a compound inequality and how is it solved?

128. Describe how to solve an absolute value inequality involving the symbol $<$. Give an example.

129. Describe how to solve an absolute value inequality involving the symbol $>$. Give an example.

130. Explain why $|x| < -4$ has no solution.

131. Describe the solution set of $|x| > -4$.

Critical Thinking Exercises

Make Sense? *In Exercises 132–135, determine whether each statement makes sense or does not make sense, and explain your reasoning.*

132. I prefer interval notation over set-builder notation because it takes less space to write solution sets.

133. I can check inequalities by substituting 0 for the variable: When 0 belongs to the solution set, I should obtain a true statement, and when 0 does not belong to the solution set, I should obtain a false statement.

134. In an inequality such as $5x + 4 < 8x - 5$, I can avoid division by a negative number depending on which side I collect the variable terms and on which side I collect the constant terms.

135. I'll win the contest if I can complete the crossword puzzle in 20 minutes plus or minus 5 minutes, so my winning time, x, is modeled by $|x - 20| \le 5$.

In Exercises 136–139, determine whether each statement is true or false. If the statement is false, make the necessary change(s) to produce a true statement.

136. $(-\infty, -1] \cap [-4, \infty) = [-4, -1]$

137. $(-\infty, 3) \cup (-\infty, -2) = (-\infty, -2)$

138. The inequality $3x > 6$ is equivalent to $2 > x$.

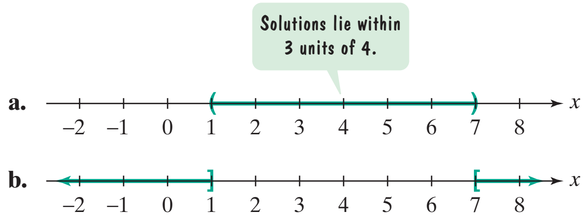
139. All irrational numbers satisfy $|x - 4| > 0$.

140. What's wrong with this argument? Suppose x and y represent two real numbers, where $x > y$.

$2 > 1$	This is a true statement.
$2(y - x) > 1(y - x)$	Multiply both sides by $y - x$.
$2y - 2x > y - x$	Use the distributive property.
$y - 2x > -x$	Subtract y from both sides.
$y > x$	Add $2x$ to both sides.

The final inequality, $y > x$, is impossible because we were initially given $x > y$.

141. Write an absolute value inequality for which the interval shown is the solution.



Group Exercise

142. Each group member should research one situation that provides two different pricing options. These can involve areas such as public transportation options (with or without discount passes), cell phone plans, long-distance telephone plans, or anything of interest. Be sure to bring in all the details for each option. At a second group meeting, select the two pricing situations that are most interesting and relevant. Using each situation, write a word problem about selecting the better of the two options. The word problem should be one that can be solved using a linear inequality. The group should turn in the two problems and their solutions.

Preview Exercises

Exercises 143–145 will help you prepare for the material covered in the first section of the next chapter.

143. If $y = 4 - x$, find the value of y that corresponds to values of x for each integer starting with -3 and ending with 3.

144. If $y = 4 - x^2$, find the value of y that corresponds to values of x for each integer starting with -3 and ending with 3.

145. If $y = |x + 1|$, find the value of y that corresponds to values of x for each integer starting with -4 and ending with 2.